

Algebra sheet-2



★ ★ ★ #

$$\text{यदि } x^n = a$$

$$\text{तो } x = a^{\frac{1}{n}}$$

$$\text{यदि } 5^{\cancel{a}} = k$$

$$\text{तो } 5 = k^{\frac{1}{a}}$$

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$



Basic →

If, $5^a = 7^b = 35^c$

~~$5^a = 7^b = 35^c = k \text{ (माना)}$~~

~~$5 = k^{\frac{1}{a}}$~~

~~$7 = k^{\frac{1}{b}}$~~

~~$35 = k^{\frac{1}{c}}$~~

तो $C = \frac{ab}{a+b}$ ✓

~~$5 \times 7 = 35$~~

~~$k^{\frac{1}{a}} \times k^{\frac{1}{b}} = k^{\frac{1}{c}}$~~

~~$\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$~~

~~$k = k$~~

✓ ⇒ ~~$\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$~~
 $\frac{b+a}{ab} = \frac{1}{c}$

Trick :->

यदि $\overset{a}{5} = \overset{b}{7} = \overset{c}{35}$

रिश्ता खोजो

$$\overset{1}{5} \times \overset{1}{7} = \overset{1}{35}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{c}$$

$$c = \frac{ab}{a+b}$$

1. **If $7^a = 11^b = 77^c$, then find value of c =?**

यदि $7^a = 11^b = 77^c$, तो c का मान ज्ञात कीजिए ?

☒ [A] $\frac{ab}{a+b}$

[C] $\frac{ab}{a-b}$

[B] $\frac{a+b}{ab}$

[D] $\frac{a+b}{a-b}$

2. If $2^x = 3^y = 72^z$, then $\frac{4xy}{2x+3y} = ?$

यदि $2^x = 3^y = 72^z$ है, तो $\frac{4xy}{2x+3y} = ?$

[A] $4z$

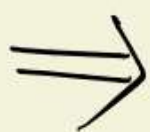
[B] $\frac{4}{z}$

[C] $2z$

[D] $\frac{3}{z}$

③ ② ①
 $2 \times 3 = 72$

$$\frac{4xy}{2x+3y} = 4z$$



$$\frac{3}{2} + \frac{2}{3} = \frac{1}{z}$$

$$\frac{3y + 2x}{6} = \frac{1}{z}$$

4. If $2^x = 3^y = 18^{-z}$, then which option is correct?

यदि $2^x = 3^y = 18^{-z}$ है, तो कौन सा विकल्प सही है?

☒ [A] $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 0$

[B] $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 0$

[C] $\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 0$

[D] $\frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 0$

$$2^1 \times 3^2 = 18^1$$

$$\frac{1}{x} + \frac{2}{y} = \frac{1}{-z}$$

$$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 0$$

3. If a, b, c are non-zero numbers and $14^a = 36^b = 84^c$, then $6b \left(\frac{1}{c} - \frac{1}{a} \right)$ is equal to?

यदि a, b, c गैर-शून्य संख्याएँ हैं और $14^a = 36^b = 84^c$ है तो $6b \left(\frac{1}{c} - \frac{1}{a} \right)$ बराबर है?

[A] 6

[C] 4

[B] 3

[D] 1.5

$$6b \times \frac{1}{2b} = 3$$

$$\Rightarrow 14^1 \times 36^{\frac{1}{2}} = 84^1$$

$$14 \times 6 = 84$$

$$6 = \sqrt{36}$$

$$\frac{1}{a} + \frac{1}{2b} = \frac{1}{c}$$

$$\frac{1}{2b} = \frac{1}{c} - \frac{1}{a}$$

5. If $(5.55)^x = (0.555)^y = 1000$, then the value of $\frac{1}{x} - \frac{1}{y}$ is

यदि $(5.55)^x = (0.555)^y = 10^3$, फिर $\frac{1}{x} - \frac{1}{y}$ का मान है

[A] 3

[B] 1

[C] $\frac{1}{3}$

[D] $\frac{2}{3}$

$$\begin{array}{rcl} 5.55 & = & \frac{555}{100} \\ \cdot 555 & = & \frac{555}{1000} \end{array}$$

$$1 = \frac{3}{x} - \frac{3}{y}$$

$$\frac{1}{3} = \frac{1}{x} - \frac{1}{y}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{10} \left(\frac{3}{x} - \frac{3}{y} \right)$$

2.

$$(ax + by)^2 + (ay - bx)^2 = (a^2 + b^2) \cdot (x^2 + y^2)$$

$$\text{L.H.S} = a^2x^2 + b^2y^2 + \cancel{2abxy} + a^2y^2 + b^2x^2 - \cancel{2abxy}$$

$$= a^2(\underline{x^2 + y^2}) + b^2(\underline{y^2 + x^2})$$

$$= (x^2 + y^2) \cdot (a^2 + b^2)$$

$$\text{L.H.S} = \text{R.H.S}$$



6. If $a^2 + b^2 = 25$, $x^2 + y^2 = 17$ and $ax + by = 8$, then $ay - bx = ?$

यदि $a^2 + b^2 = 25$, $x^2 + y^2 = 17$ और $ax + by = 8$ है, तो $ay - bx$ ज्ञात कीजिए?

(SSC CHSL PRE 2020)

[A] 20

[B] 17

[C] 18

~~[D] 19~~

$$(ax + by)^2 + (ay - bx)^2 = (a^2 + b^2) \cdot (x^2 + y^2)$$

$$8^2 + ?^2 = 25 \times 17$$

$$?^2 = 425 - 64 = 361$$

$$? = 19$$

③

If, $a \times b = 1$ तो $\frac{1}{1+a^n} + \frac{1}{1+b^n} = 1$

$$\rightarrow b = \frac{1}{a}$$

$$\begin{aligned} & \frac{1}{1+a^n} + \frac{1}{1+\frac{1}{a^n}} \\ & \frac{1}{1+a^n} + \frac{a^n}{a^n+1} \\ & = \frac{\cancel{1+a^n}}{\cancel{1+a^n}} = 1 \end{aligned}$$

if, $a = \frac{\sqrt{10} + \sqrt{7}}{\sqrt{10} - \sqrt{7}}$

$$b = \frac{\sqrt{10} - \sqrt{7}}{\sqrt{10} + \sqrt{7}}$$

$$a \times b = 1$$

तो $\frac{1}{1+a^2} + \frac{1}{1+b^2} = ①$



$$\text{If, } a = 5 + 2\sqrt{6}$$

$$b = 5 - 2\sqrt{6}$$

$$\begin{aligned} a \times b &= 5^2 - (2\sqrt{6})^2 \\ &= 25 - 24 \\ &= 1 \end{aligned}$$

$$\text{तो } \frac{1}{1+a^{37}} + \frac{1}{1+b^{37}} = 2$$



7. If $P + \frac{1}{p} = 112$, then the value of $(p - 112)^{15} + \left(\frac{1}{p}\right)^{15}$ will be:

यदि $P + \frac{1}{p} = 112$ है, तो $(p - 112)^{15} + \left(\frac{1}{p}\right)^{15}$ का मान होगा:

[A] 0

[C] 112

[B] 1

[D] -1

≠

$$p - 112 = -\frac{1}{p}$$



$$\left(-\frac{1}{p}\right)^{15} + \left(\frac{1}{p}\right)^{15} = 0$$

$$\begin{cases} n = 1 \\ (-1)^{\text{odd}} = -1 \\ (-1)^{\text{even}} = 1 \end{cases}$$

8. If $x + \frac{4\sqrt{3}}{x} = 5$, then $\frac{x^2 + 7x + 4\sqrt{3}}{x^2(5-x)} = ?$

यदि $(x + \frac{4\sqrt{3}}{x} = 5)$ है, तो $\frac{x^2 + 7x + 4\sqrt{3}}{x^2(5-x)} = ?$

[A] $\sqrt{3}$

[B] $2\sqrt{3}$

[C] $3\sqrt{3}$

[D] 2

$$x^2 + 4\sqrt{3} = 5x$$

$$4\sqrt{3} = 5x - x^2$$

$$\frac{12}{x \cdot (5-x)}$$

$$= \frac{12}{5x - x^2} = \frac{12}{4\sqrt{3}} = \sqrt{3}$$

11. If $\frac{11-13x}{x} + \frac{11-13y}{y} + \frac{11-13z}{z} = 5$, then what is the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$?

यदि $\frac{11-13x}{x} + \frac{11-13y}{y} + \frac{11-13z}{z} = 5$, तो $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ का मान क्या है?

[A] 5

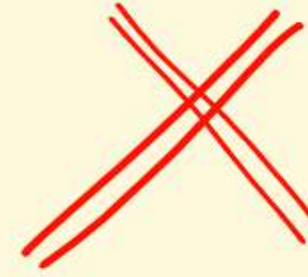
[B] 13/11

[C] 13/5

[D] 4.

$$11\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 4$$

$$\frac{a+b-c}{x} = \frac{a}{x} + \frac{b}{x} - \frac{c}{x}$$



9. If $(3x + 4y)(5x - 6y) = ax^2 + 2hxy + by^2$, then what is the value of $(4a - h + 2b)$?

यदि $(3x + 4y)(5x - 6y) = ax^2 + 2hxy + by^2$ है, तो $(4a - h + 2b)$ का मान कितना होगा?

[A] 13

[B] 107

[C] 108

[D] 11

$$60 - 1 - 48 = 11$$

$$a = 15$$

$$2h = 2 \Rightarrow h = 1$$

$$b = -24$$

10. If $(2x + 3y + 4)(2x + 3y - 5)$ is equivalent to $(ax^2 + by^2 + 2hxy + 2gx + 2fy + c)$, then what is the value of $\{(g + f - c)/abh\}$?

यदि $(2x + 3y + 4)(2x + 3y - 5)$, $(ax^2 + by^2 + 2hxy + 2gx + 2fy + c)$ के समतुल्य है, तो $\{(g + f - c)/abh\}$ का मान ज्ञात कीजिए।

[A] $\frac{37}{216}$
[C] $\frac{19}{108}$

[B] $\frac{19}{216}$
[D] $\frac{35}{432}$

$$\frac{-1 - \frac{3}{2} + 20}{4 \times 9 \times 6}$$

$$a = 4$$

$$b = 9$$

$$2h = 12$$

$$c = -20$$

$$g = -1$$

$$f = -3$$

$$= \frac{35}{216}$$

$$\frac{19 - 3}{2}$$

If, $x + \frac{1}{x} = c + \frac{1}{c}$

तो, $x = c, \frac{1}{c}$

या

$$x + \frac{1}{x} = \left(\frac{1}{c}\right) + \left(\frac{1}{c}\right)$$

#5

$$c = \frac{1}{\left(\frac{1}{c}\right)}$$

If, $x + \frac{1}{x} = 7 + \frac{1}{7}$

तो $x = 7, \frac{1}{7}$

12. **If $x + \frac{1}{x} = c + \frac{1}{c}$ then the value of x ?**

यदि $x + \frac{1}{x} = c + \frac{1}{c}$ तो x का मान बताइए?

(SSC CGL Mains 2016)

☒ [A] $c, \frac{1}{c}$

[B] c, c^2

[C] $c, 2c$

[D] $0, 1$

$$\frac{a+b}{a-b} + \frac{a-b}{a+b} = \frac{2(a^2 + b^2)}{(a^2 - b^2)} \checkmark$$

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Proof:

$$\frac{a+b}{a-b} + \frac{a-b}{a+b} = \frac{(a+b)^2 + (a-b)^2}{(a-b) \cdot (a+b)} = \frac{2(a^2 + b^2)}{(a^2 - b^2)}$$

$$\frac{\sqrt{11} + \sqrt{7}}{\sqrt{11} - \sqrt{7}} + \frac{\sqrt{11} - \sqrt{7}}{\sqrt{11} + \sqrt{7}} = \frac{2 \times (11 + 7)}{11 - 7} = \frac{36}{4} = 9$$



✗

$$\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{4ab}{a^2 - b^2}$$

$$\frac{a-b}{a+b} - \frac{a+b}{a-b} = \frac{-4ab}{a^2 - b^2}$$

$$\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}} - \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} = -\frac{4 \times \sqrt{7} \times \sqrt{3}}{7-3} = -\sqrt{21}$$



13. If $x^{150} = \frac{6-\sqrt{21}}{6+\sqrt{21}}$, then $x^{150} + \frac{1}{x^{150}} = ?$

यदि $x^{150} = \frac{6-\sqrt{21}}{6+\sqrt{21}}$ हो तो, $x^{150} + \frac{1}{x^{150}} = ?$

[A] 8.7

[B] 8.6

[C] 6.7

[D] 7.6

$$\frac{6-\sqrt{21}}{6+\sqrt{21}} + \frac{6+\sqrt{21}}{6-\sqrt{21}}$$

$$= \frac{2 \times (36+21)}{(36-21)} = 2 \times \frac{57}{15} = \frac{38}{5}$$

14. If the value of $\frac{3x\sqrt{y}+2y\sqrt{x}}{3x\sqrt{y}-2y\sqrt{x}} - \frac{3x\sqrt{y}-2y\sqrt{x}}{3x\sqrt{y}+2y\sqrt{x}}$ is same as that of $\sqrt{x} \sqrt{y}$

following relations:

$\frac{3x\sqrt{y}+2y\sqrt{x}}{3x\sqrt{y}-2y\sqrt{x}} - \frac{3x\sqrt{y}-2y\sqrt{x}}{3x\sqrt{y}+2y\sqrt{x}}$ का मान $\sqrt{x} \sqrt{y}$ के मान के बराबर है तो x और y के बीच निम्न

[A] $9x - 4y = 36$

[C] $9x + 4y = 36$

[B] $9x + 4y = 24$

[D] $9x - 4y = 24$

$a \rightarrow 3x\sqrt{y}$

$b \rightarrow 2y\sqrt{x}$

$$\frac{4 \times \cancel{3x\sqrt{y}} \times \cancel{2y\sqrt{x}}}{x^2y - 4y^2x} = \cancel{\sqrt{x}\sqrt{y}}$$

$$\frac{\cancel{24xy}}{xy(x-4y)} = 1$$

16. Let $x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $y = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ What is $(x^2 - y^2)$ equal to?

यदि $x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $y = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ तो $(x^2 - y^2)$ किसके बराबर है?

[A] $22\sqrt{30}$

[B] $44\sqrt{30}$

[C] $66\sqrt{30}$

[D] $88\sqrt{30}$

$(x+y) \times (x-y)$

$\frac{2 \times (6+5)}{1} \times \frac{4 \times \sqrt{6} \times \sqrt{5}}{1}$

$= 88\sqrt{30}$

15. If $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then find the value of $x^2 - y^2$.

यदि $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ और $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$ है, तो $x^2 - y^2$ का मान ज्ञात करें।

[A] $\sqrt{5}$

[B] $2\sqrt{5}$

[C] $3\sqrt{5}$

[D] $4\sqrt{5}$

$$\frac{3}{\cancel{2 \times 6} \cancel{4}} \times \frac{\cancel{4} \sqrt{5}}{\cancel{4}}$$

17. If $x = \sqrt{6} + 2$ and $y = \sqrt{6} - 2$, then what is the value of $\left(\frac{x}{y} + \frac{y}{x}\right)^2 - 3 = ?$

यदि $x = \sqrt{6} + 2$ और $y = \sqrt{6} - 2$, तो $\left(\frac{x}{y} + \frac{y}{x}\right)^2 - 3$ का मान क्या होगा?

[A] 42

[C] 35

[B] 97 = 10² - 3

[D] 22

$$\frac{\sqrt{6}+2}{\sqrt{6}-2} + \frac{\sqrt{6}-2}{\sqrt{6}+2} = \frac{\cancel{2} \times (6+4)}{\cancel{6}-4} = 10$$