

Algebra - 5th sheet



If $a \in \text{real No.}$

$$\text{तो } a^2 \geq 0$$

$$a^2 \neq -ve$$

$$(5)^2 = (-5)^2 = 25$$

$$\left. \begin{array}{l} (+ve)^2 = +ve \\ (-ve)^2 = +ve \\ (0)^2 = 0 \end{array} \right\}$$

multiple variables from
single equation

Special Case

x, y & z
 $\in$ real No's.

$$x^2 + y^2 + z^2 = 0$$

$$0 \quad 0 \quad 0$$

$$\Rightarrow x = y = z = 0$$

Ex $\Rightarrow$

$$(x - 2)^2 + (y - 3)^2 + (z - 5)^2 = 0$$

$$0$$

$$x - 2 = 0$$

$$x = 2$$

$$0$$

$$y - 3 = 0$$

$$y = 3$$

$$0$$

$$z - 5 = 0$$

$$z = 5$$



1.

If $(a-11)^2 + (b-12)^2 + (c+7)^2 = 0$ then $\sqrt{a+b+c} = ?$

यदि $(a-11)^2 + (b-12)^2 + (c+7)^2 = 0$, तो $\sqrt{a+b+c} = ?$

[A] 2

[C] ± 4

[B] 3

[B] 4 ✓

$$\sqrt{16} = 4$$

$$a = 11$$

$$b = 12$$

$$c = -7$$

2. If $(a+b-c-7)^2 + (b+c-a-10)^2 + (c+a-b-19)^2 = 0$, then $\sqrt{a+b+c} = ?$

यदि $(a+b-c-7)^2 + (b+c-a-10)^2 + (c+a-b-19)^2 = 0$, तो $\sqrt{a+b+c} = ?$

[A] $2\sqrt{2}$



[B] 5



[C] 6

[D] $\sqrt{2}$

= 6

$$\cancel{a} + b - \cancel{c} = 7$$

$$\cancel{b} + \cancel{c} - \cancel{a} = 10$$

$$c + a - \cancel{b} = 19$$

all

$$a + b + c = 36$$

3. If $(3x - 4y)^2 + (5y - 2z)^2 = 0$, then find $x : y : z = ?$

यदि $(3x - 4y)^2 + (5y - 2z)^2 = 0$, तो $x : y : z$ ज्ञात कीजिए।

[A] 8:6:15

[B] 4:3:5

[C] 8:3:5

[D] 8:3:15

$$3x = 4y$$

$$5y = 2z$$

x	y	z
4	3	
	↓	
	2	5
$8 : 6 : 15$		

4. If $(r \cos \theta - \sqrt{3})^2 + (r \sin \theta - 1)^2 = 0$, Then $\frac{r \tan \theta + \sec \theta}{r \sec \theta + \tan \theta} = ?$

यदि $(r \cos \theta - \sqrt{3})^2 + (r \sin \theta - 1)^2 = 0$, तो $\frac{r \tan \theta + \sec \theta}{r \sec \theta + \tan \theta} = ?$

[A] $\frac{4}{5}$

[B] $\frac{3}{5}$

[C] $\frac{\sqrt{5}}{4}$

[D] $\frac{\sqrt{3}}{4}$

$$\frac{\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{\frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{4}{5}$$

$\div$

$$r \cos \theta = \sqrt{3} \Rightarrow r \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$r \sin \theta = 1$$

$$\cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

$$r = 2$$

5. If $(x - y + 5)^2 + (x^2 + xy + y^2 - 8)^2 = 0$, then find $x^3 - y^3$?

यदि $(x - y + 5)^2 + (x^2 + xy + y^2 - 8)^2 = 0$ है, तो $x^3 - y^3$ ज्ञात कीजिये?

[A] -56

[B] 40

[C] 35

[D] -40

$$\begin{array}{cc} x - y = -5 & \times \\ \times & \times \\ x^2 + xy + y^2 = 8 & \end{array}$$

6. If $\left(x^3 + \frac{1}{x^3} - k\right)^2 + \left(x + \frac{1}{x} - p\right)^2 + \left(x^2 + \frac{1}{x^2} - t\right)^2 = 0$, where k, p and t are real numbers and $x \neq 0$, then $\frac{k}{p}$ is equal to:

यदि $\left(x^3 + \frac{1}{x^3} - k\right)^2 + \left(x + \frac{1}{x} - p\right)^2 + \left(x^2 + \frac{1}{x^2} - t\right)^2 = 0$ हो, जहाँ k, p और t वास्तविक संख्याएँ हैं और $x \neq 0$ है, तो $\frac{k}{p}$ किसके बराबर होगा?

[A] ~~t~~

[B] $t+1$

[C] $t-1$

[D] $t-2$

$$\frac{k}{p} = \frac{p^3 - 3p}{p} = (p^2 - 3) - 1 = t - 1$$

$$x^3 + \frac{1}{x^3} = k = p^3 - 3p$$

$$x + \frac{1}{x} = p$$

$$x^2 + \frac{1}{x^2} = t = p^2 - 2$$

7. If $x^2 + y^2 - 10x + 26y + 194 = 0$, then the value of $x^2 + y^2$ is:
यदि $x^2 + y^2 - 10x + 26y + 194 = 0$, तो $x^2 + y^2$ का मान ज्ञात कीजिए।

[A] 190

[B] 194

$25 + 169$

[C] 198

[D] 200

$$(x^2 - 10x + 25) + (y^2 + 26y + 169) = 0$$

$$(x - 5)^2 + (y + 13)^2 = 0$$

$\downarrow$ $\downarrow$
 $x = 5$ $y = -13$

8. If $x^2 + y^2 + z^2 = 2(5x + 6y + 4z) - 77$ then find the value of $2(x + y) - 3z$.

अगर $x^2 + y^2 + z^2 = 2(5x + 6y + 4z) - 77$ तो $2(x + y) - 3z$ का मान ज्ञात कीजिए।

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[A] 11

[B] 78

[C] 10

[D] 23

22-12

~~$$(x^2 - 10x + 25) + (y^2 - 12y + 36) + (z^2 - 8z + 16) = 0$$~~

~~$$(x-5)^2 + (y-6)^2 + (z-4)^2 = 0$$~~

$x=5$

$y=6$

$z=4$

8. If $x^2 + y^2 + z^2 = 2(5x + 6y + 4z) - 77$ then find the value of $2(x + y) - 3z$.
 अगर $x^2 + y^2 + z^2 = 2(5x + 6y + 4z) - 77$,तो $2(x + y) - 3z$ का मान ज्ञात कीजिए।

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[A] 11

[B] 78

[C] 10

[D] 23

$$x^2 + y^2 + z^2 = 2(5x + 6y + 4z) - 77$$

$$x = 5$$

$$y = 6$$

$$z = 4$$

9. If $a^2 + b^2 + c^2 + 216 = 12(a + b - 2c)$, then $\sqrt{ab - bc - ca}$ is:

यदि $a^2 + b^2 + c^2 + 216 = 12(a + b - 2c)$ है, तो $\sqrt{ab - bc - ca}$ का मान ज्ञात करें।

[A] $6\sqrt{5}$

[B] $4\sqrt{5}$

[C] $3\sqrt{5}$

[D] $8\sqrt{5}$

$$\sqrt{36 + 72 + 72} = \sqrt{180}$$

$$a^2 + b^2 + c^2 + 216 = 12(a + b - 2c)$$

$$a = 6$$

$$b = 6$$

$$c = -12$$

10. If $16x^2 + 9y^2 + 4z^2 = 24(x - y + z) - 61$, then the value of $(xy + 2z)$ is:

यदि $16x^2 + 9y^2 + 4z^2 = 24(x - y + z) - 61$ है, तो $(xy + 2z)$ का मान है:

[A] 1

[B] 2

[C] 3

[D] 5 ✓

$$\cancel{\frac{3}{4}}x - \cancel{\frac{4}{3}} + 6 = 5$$

$$\textcircled{16}x^2 + \textcircled{9}y^2 + \textcircled{4}z^2 = 24 \cdot (\textcircled{1}x - \textcircled{1}y + \textcircled{1}z) - 61$$

$$x = \frac{\cancel{12}}{\cancel{16}} = \frac{3}{4}$$

$$y = -\frac{\cancel{12}}{\cancel{9}} = -\frac{4}{3}$$

$$z = \frac{\cancel{12}}{\cancel{4}} = 3$$

11. If $x^2 + 9y^2 + 4z^2 = 12(x - 2y + 2z) - 88$, then the value of $(x - 3y + z)$ is:

यदि $x^2 + 9y^2 + 4z^2 = 12(x - 2y + 2z) - 88$ है, तो $(x - 3y + z)$ का मान ज्ञात करें।

[A] 10

[B] 13 ✓

[C] 11

[D] 5



12. If $4(x^2 + z) + y^2 + z^2 = 4x + 10(y - 3)$, then find $(2x + 3y + z)$?

यदि $4(x^2 + z) + y^2 + z^2 = 4x + 10(y - 3)$ है, तो $(2x + 3y + z)$ ज्ञात कीजिये?

[A] 14

[B] 13

$1 + 15 - 2$

[C] 12

[D] 10

$$4x^2 + y^2 + z^2 = 2 \cdot (2x + 5y - 2z) - 30$$

$$x = \frac{1}{2}$$

$$y = 5$$

$$z = -2$$

13. If $3x(3x-8) + y(y+1) + 4z^2 - 2z + 16\frac{1}{2} = 0$, then what is the value of $(6x + y + 2z)$?

यदि $3x(3x-8) + y(y+1) + 4z^2 - 2z + 16\frac{1}{2} = 0$ है, तो $(6x+y+2z)$ का मान क्या है?

[A] 2

[B] 6

[C] 8

[D] 4

$$8 - \frac{1}{2} + \frac{1}{2}$$

$$9x^2 + y^2 + 4z^2 + 16\frac{1}{2} = 24x - y + 2z$$

$$= 2 \cdot \left(\frac{12x}{2} - \frac{1}{2}y + \frac{1}{2}z \right)$$

$$x = \frac{12}{3} = 4$$

$$y = -\frac{1}{2}$$

$$z = \frac{1}{4}$$



Test



14. If $10x^2 + y^2 + 6xy + 2x + 1 = 0$, then $3x+4y=?$

यदि $10x^2 + y^2 + 6xy + 2x + 1 = 0$ है, तो $3x+4y=?$

[A] 9 ✓

[B] 12

[C] 10

[D] 7

$$-3 + 12 = 9$$

$$(x^2 + 2x + 1) + (9x^2 + y^2 + 6xy) = 0$$

$$(x+1)^2 + (3x+y)^2 = 0$$

$\downarrow \qquad \qquad \downarrow$
 $x = -1 \qquad y = -3x = 3$

16. If $9x^2 + 41y^2 + 4z^2 = 24xy + 20yz$, then find $x:y:z = ?$

यदि $9x^2 + 41y^2 + 4z^2 = 24xy + 20yz$ है, तो ज्ञात कीजिए कि $x:y:z = ?$

[A] ~~3:2:5~~

[B] ~~8:6:15~~

[C] ~~16:12:25~~

[D] ~~6:8:15~~

$\begin{matrix} 4 & 3 \\ 2 & 5 \end{matrix}$

$$(9x^2 - 24xy + 16y^2) + (25y^2 - 20yz + 4z^2) = 0$$

$$(3x - 4y)^2 + (5y - 2z)^2 = 0$$

$\downarrow$
0
 $3x = 4y$

$\downarrow$
0
 $5y = 2z$

15. If $a^8 + 9 = \frac{a^4}{b^2} [10b^2 - b^4 - 4]$, then $a^{12} + b^{10} = ?$

यदि $a^8 + 9 = \frac{a^4}{b^2} [10b^2 - b^4 - 4]$, तो $a^{12} + b^{10} = ?$

[A] 59

[B] 63

[C] 48

[D] 77

$$= (a^4)^3 + (b^2)^5 \\ = 27 + 32$$

$$a^4 + \frac{9}{a^4} = 10 - b^2 - \frac{4}{b^2}$$

$$\left(a^4 + \frac{9}{a^4} - 6 \right) + \left(b^2 + \frac{4}{b^2} - 4 \right) = 0$$

$$\left(a^2 - \frac{3}{a^2} \right)^2 + \left(b - \frac{2}{b} \right)^2 = 0$$

$$\downarrow \qquad \qquad \downarrow$$

$$a^2 = \frac{3}{a^2} \qquad b = \frac{2}{b}$$

$$a^4 = 3 \qquad b^2 = 2$$



(imp)

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$$

$$(a - b + c)^2 = a^2 + b^2 + c^2 + 2(-ab - bc + ca)$$

$$(-a + b + c)^2 = a^2 + b^2 + c^2 + 2(-ab + bc - ca)$$

$$x^2 = (-x)^2$$

$$(-a+b-c)^2 = (a-b+c)^2$$

$$(a + b + c)^2 = \underbrace{(a^2 + b^2 + c^2)}_{?} + 2 \underbrace{(ab + bc + ca)}_{??}$$

$$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$$

$$(ab + bc + ca) = \frac{(a + b + c)^2 - (a^2 + b^2 + c^2)}{2}$$

17. Write the expanded form of $(8a + 5b + 9c)^2$.

$(8a + 5b + 9c)^2$ को विस्तारित रूप में लिखिए।

[RRB JE 2024]

[A] $64a^2 + 25b^2 + 81c^2 + 84ab + 90bc + 144ac$

[B] $64a^2 + 25b^2 + 81c^2 + 80ab + 85bc + 144ac$

[C] $64a^2 + 25b^2 + 81c^2 + 80ab + 90bc + 144ac$ ✓

[D] $64a^2 + 25b^2 + 81c^2 + 80ab + 90bc + 154ac$

18. If $(a + b + c) = 7$ and $ab + bc + ca = 12$, find the value of $a^2 + b^2 + c^2$.

यदि $(a + b + c) = 7$ and $ab + bc + ca = 12$, तो $a^2 + b^2 + c^2$ का मान ज्ञात कीजिए।

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[A] 29

[C] 27

[B] 31

[D] 25 ✓

$$= 7^2 - 2 \times 12$$



19. If $x + y + z = 16$ and $xy + yz + zx = 75$, what is the value of $x^2 + y^2 + z^2$?

यदि $x + y + z = 16$ और $xy + yz + zx = 75$, तो $x^2 + y^2 + z^2$ का मान क्या है?

(SSC CHSL PRE 2025)

[A] 106 ✓

[C] 156

[B] 121

[D] 108

$$= (x + y + z)^2 - 2(xy + yz + zx)$$

$$= 256 - 2 \times 75$$

$$= 106$$

20. If $a + b + c = 0$ and $ab + bc + ca = -11$, then what is the value of $a^2 + b^2 + c^2$?

यदि $a + b + c = 0$ और $ab + bc + ca = -11$, तो $a^2 + b^2 + c^2$ का मान क्या है?

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[A] -11

[C] 0

[B] 22 ✓

[D] 11

$$= 0^2 - 2 \times (-11)$$



Tc



21. If $(a + b + c)/2 = 16$ and $2ab + 2bc + 2ca = 120$, then what is the value of $4a^2 + 4b^2 + 4c^2$?
यदि $(a + b + c)/2 = 16$ and $2ab + 2bc + 2ca = 120$, तो $4a^2 + 4b^2 + 4c^2$ का मान क्या है?

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[A] 2828

[B] 3368

[C] 3616

[D] 3056

$$= 4 \times [32^2 - 2 \times 60]$$

$$= 4096 - 480$$

$$a + b + c = 32$$

$$ab + bc + ca = 60$$

22. If $(x + y + z) = 23$ and $x^2 + y^2 + z^2 = 179$, then find the value of $(xy + yz + zx)$.

यदि $(x + y + z) = 23$ और $x^2 + y^2 + z^2 = 179$ है, तो $(xy + yz + zx)$ का मान ज्ञात कीजिए।

(SSC SELECTION POST XII HIGHER SECONDARY LEVEL)

[A] 225

[B] 280

[C] 175

[D] 350

$$\frac{(x+y+z)^2 - (x^2 + y^2 + z^2)}{2}$$

$$= \frac{529 - 179}{2}$$

$$= 175$$



23. **a, b, c are three positive numbers, such that $(a + b + c) = 20$, $a^2 + b^2 + c^2 = 152$.
The value of $(ab + bc + ca)$ is equal to:**

**a, b, c तीन धनात्मक संख्याएं हैं इस प्रकार हैं कि $(a + b + c) = 20$, $a^2 + b^2 + c^2 = 152$ हैं।
 $(ab + bc + ca)$ का मान बराबर है:**

[A] 124

[B] 110

[C] 112

[D] 102

→
$$\frac{20^2 - 152}{2}$$
$$= \frac{248}{2}$$

24. If $x + y + z = 0$ and $x^2 + y^2 + z^2 = 32$ then find the value of, $xy + yz + xz$.

यदि $x + y + z = 0$, और $x^2 + y^2 + z^2 = 32$, तो $xy + yz + xz$ का मान ज्ञात कीजिए।

SSC CHSL PRE 2025

[A] -16 ✓

[B] 16

[C] -32

[D] 32

$$= \frac{0^2 - 32}{2}$$

25. If $a^2 + b^2 + c^2 = 271$ and $ab + bc + ca = 45$, then what is the value of $a + b + c$?

यदि $a^2 + b^2 + c^2 = 271$, और $ab + bc + ca = 45$ है, तो $a + b + c$ का मान क्या है?

[A] 20

[B] 22

$= \pm 19$

[C] 19 ✓

[D] 17

$$(a+b+c)^2 = 271 + 2 \times 45$$

$$= 361$$

26. If $a^2+b^2+c^2 = 6.25$ and $(ab+bc+ca) = 0.52$, what is the value of $(a+b+c)$, if $(a+b+c) < 0$?
यदि $a^2+b^2+c^2 = 6.25$ और $(ab+bc+ca) = 0.52$ है, तो $(a+b+c)$ का मान ज्ञात करें, यदि $(a+b+c) < 0$ है?

$$= -2.7$$

[A] ~~± 2.7~~

[B] -2.7

[C] -2.8

[D] ~~± 2.8~~

$$\begin{aligned}(a+b+c)^2 &= 6.25 + 2 \times 0.52 \\ &= 7.29\end{aligned}$$

27. If $a + b + c = 6$ & $(a^2 + b^2 + c^2) = 14$ & $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{6}$, then find abc ?

यदि $a + b + c = 6$ & $(a^2 + b^2 + c^2) = 14$ & $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{6}$ है, तो abc ज्ञात कीजिये?

[A] 12

[B] 6

[C] 1

[D] 11

$$\frac{6^2 - 14}{2} = 11$$

$$\frac{bc + ca + ab}{abc} = \frac{11}{6}$$