



यदि $x^4 + x^2 y^2 + y^4 = 189$, $x^2 + xy + y^2 = 21$

(i). $x^2 - xy + y^2 = \frac{189}{21} \Rightarrow 9$

(vi) $x + y = ?$

$(x^2 + xy + y^2) + xy = (21) + 6$

$(x + y)^2 = 27 \therefore x + y = 3\sqrt{3}$

(ii). $x^2 + y^2 = \frac{21+9}{2} \Rightarrow 15$

(iii) $xy = \frac{21-9}{2} \Rightarrow 6$

(iv) $\frac{x}{y} + \frac{y}{x} = \frac{x^2 + y^2}{xy} \Rightarrow \frac{21+9}{21-9} \Rightarrow \frac{5}{2}$

(v) $\frac{x^2}{y^2} + \frac{y^2}{x^2} = \left(\frac{5}{2}\right)^2 - 2 \Rightarrow \frac{17}{4}$

$(x+y)^2 = x^2 + y^2 + 2xy$

Handwritten notes on the right side of the page showing calculations for $x^2 + y^2$ and xy using the given equations.

घात 2 और घात 4 के बीच संबंध (Relation between Power 2 and Power 4)



$$\Rightarrow (x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$$

$$\Rightarrow (x^2 + y^2)^2 - x^2y^2 = x^4 + y^4 + x^2y^2$$

$$\Rightarrow (x^2 + y^2) - (xy)^2 = x^4 + y^4 + x^2y^2$$

A

B

$$\Rightarrow (x^2 + y^2 + xy)(x^2 + y^2 - xy) = x^4 + y^4 + x^2y^2$$

$$\Rightarrow x^2 + y^2 + xy = A \text{ और } x^2 + y^2 - xy = B$$

$$\Rightarrow x^2 + y^2 = \frac{A+B}{2} \quad xy = \frac{A-B}{2}$$



49. If $x^2 - xy + y^2 = 7$ and $x^4 + x^2y^2 + y^4 = 105$, then the value of $\frac{x^3}{y^3} + \frac{y^3}{x^3}$ is?

यदि $x^2 - xy + y^2 = 7$ और $x^4 + x^2y^2 + y^4 = 105$ तो $\frac{x^3}{y^3} + \frac{y^3}{x^3}$ का मान क्या है?

(a) $\frac{437}{32}$
(c) $\frac{713}{64}$

(b) $\frac{317}{16}$
(d) $\frac{803}{64}$

$$= k \cdot (k^2 - 3)$$
$$= \frac{11}{4} \cdot \left(\frac{121}{16} - 3 \right)$$

$$x^2 + xy + y^2 = 15 \quad A$$

$$x^2 - xy + y^2 = 7 \quad B$$

$$\frac{x}{y} + \frac{y}{x} = \frac{15+7}{15-7} = \frac{11}{4}$$



50. If $x^4 + y^4 + x^2y^2 = 17\frac{1}{16}$ and $x^2 - xy + y^2 = 5\frac{1}{4}$, then one of the values of $(x - y)$ is:

यदि $x^4 + y^4 + x^2y^2 = 17\frac{1}{16}$ और $x^2 - xy + y^2 = 5\frac{1}{4}$ तो $(x - y)$ का मान है:

(a) $\frac{5}{2}$
(c) $\frac{5}{4}$

$= \frac{273}{16}$ (b) $\frac{3}{4}$
(d) $\frac{3}{2}$

$$x^2 + xy + y^2 = \frac{13}{4}$$

$$x^2 - xy + y^2 = \frac{21}{4} + 1$$

$$(x - y)^2 = \frac{25}{4}$$

$$xy = \frac{\frac{13}{4} - \frac{21}{4}}{2} = -1$$

51. The value of $\frac{(4.6)^4 + (5.4)^4 + (24.84)^2}{(4.6)^2 + (5.4)^2 + 24.84}$ is:

$\frac{(4.6)^4 + (5.4)^4 + (24.84)^2}{(4.6)^2 + (5.4)^2 + 24.84}$ का मान क्या है?

(a) 24.42

(b) 25.48 ✓

(c) 24.24

(d) 25.42

$$= 4.6^2 + 5.4^2 + 24.84$$

$$= \dots 6 + \dots 6 + \dots 4$$



52. If $\frac{81m^4 + 256n^4 + 144m^2n^2}{(9m^2 + 16n^2 + 12mn)} = [A\sqrt{3}m^2 + B\sqrt{2}n^2 - C\sqrt{3}mn]$, then the value of $A^2 + B^2 - C^2$ is.

यदि $\frac{81m^4 + 256n^4 + 144m^2n^2}{(9m^2 + 16n^2 + 12mn)} = [A\sqrt{3}m^2 + B\sqrt{2}n^2 - C\sqrt{3}mn]$, तो $A^2 + B^2 - C^2$ का मान है।

(a) 127

(c) 117

(b) 107

(d) 137

$$27 + 128 - 48 = 107$$

$$= 9m^2 + 16n^2 - 12mn$$

$$x \rightarrow 3m \\ y \rightarrow 4n$$

$$A = 3\sqrt{3}$$

$$B = 8\sqrt{2}$$

$$C = 4\sqrt{3}$$



53. If $9x^4 + 20x^2y^2 + 16y^4 = 91$ and $3x^2 + 2xy + 4y^2 = 13$, then find the value of $\frac{x}{4y} + \frac{y}{3x} = ?$

यदि $9x^4 + 20x^2y^2 + 16y^4 = 91$ और, $3x^2 + 2xy + 4y^2 = 13$ तो $\frac{x}{4y} + \frac{y}{3x}$ का मान ज्ञात करें?

- (a) $\frac{7}{12}$
(c) $\frac{2}{3}$

- (b) 0.625
(d) $\frac{5}{9}$

$3x^2 + 2xy + 4y^2 = 13$
 $3x^2 - 2xy + 4y^2 = 7$

$= \frac{3x^2 + 4y^2}{12xy} \rightarrow \frac{10}{18}$

$4xy = 6$



54. If $a^4 + b^4 = 63$ & $a^2 + b^2 + ab\sqrt{2} = 9$, then find ab ?

यदि $a^4 + b^4 = 63$ & $a^2 + b^2 + ab\sqrt{2} = 9$ है, तो ab ज्ञात कीजिये?

(a) 1

☒ (b) $1/\sqrt{2}$

(c) 2

(d) 0

$$a^2 + b^2 - ab\sqrt{2} = 7$$

$$ab\sqrt{2} = 1$$



~~ae~~
ae → ae
y → 1

$$x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1) \checkmark$$

$$(x^2 + y^2)^2 = \underline{x^4 + y^4} + 2x^2y^2$$

$$(x^2 + y^2)^2 - 2x^2y^2$$

$$x^4 + y^4 =$$

$$(x^2 + y^2)^2 - (\sqrt{2}xy)^2$$

$$\underline{ae^4 + y^4}$$

$$(x^2 + y^2 + \sqrt{2}xy)(x^2 + y^2 - \sqrt{2}xy)$$



55. If $\sqrt{\frac{a}{b}} = \frac{8}{3} + \sqrt{\frac{b}{a}}$ & $(a + b) = 30$, then what is the value of ab ?

यदि $\sqrt{\frac{a}{b}} = \frac{8}{3} + \sqrt{\frac{b}{a}}$ & $(a + b) = 30$, तो ab का मान क्या है?

(a) 64

(d) 81

(b) 28

(d) 26

$$\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}} = \frac{8}{3}$$

$$x - \frac{1}{x} = 3 - \frac{1}{3}$$

$$\sqrt{\frac{a}{b}} = 3 \Rightarrow \frac{a}{b} = \frac{9}{1}$$



56. If $(x+5)(x+6) + \frac{9}{x(x-1)} = 0$, then $2x^2 + 10x + 7 = ?$

यदि $(x+5)(x+6) + \frac{9}{x(x-1)} = 0$, तो $2x^2 + 10x + 7 = ?$

(a) 10

(c) 13

(b) 11

(d) 17

$$x(x-1)(x+5)(x+6) + 9 = 0$$

$$(x^2 + 5x)(x^2 + 5x - 6) + 9 = 0$$

$$y(y-6) + 9 = 0$$

$$y^2 - 6y + 9 = 0 \Rightarrow (y-3)^2 = 0$$
$$y = 3$$

$$= 2(x^2 + 5x) + 7$$
$$= 2 \times 3 + 7$$

$$\text{माना } x^2 + 5x = y$$



57. Given that x, y, z are positive real numbers, if $(x + y)^2 - z^2 = 130$, $(y + z)^2 - x^2 = 150$ and $(x + z)^2 - y^2 = 120$, then x is:

दिया गया है कि x, y, z धनात्मक वास्तविक संख्याएं हैं, यदि $(x+y)^2-z^2=130$, $(y+z)^2-x^2=150$ और $(x+z)^2-y^2=120$ है, तो x किसके समान होगा?

(a) $\frac{35}{8}$
(c) $\frac{15}{2}$

(CHSL Pre 2020)

(b) $\frac{25}{4}$
(d) $\frac{16}{3}$

$$x + y + z = 20$$

$$y + z - x = \frac{15}{2}$$

$$2x = \frac{25}{2}$$

$$(x+y+z) \cdot (x+y-z) = 130$$

$$(y+z+x) \cdot (y+z-x) = 150$$

$$(x+z+y) \cdot (x+z-y) = 120$$

+

$$(x+y+z)^2 = 400$$

$$x+y+z = 20$$



58. **If $\frac{ab}{a+b} = \frac{1}{3}$, $\frac{bc}{b+c} = \frac{1}{4}$, $\frac{ca}{c+a} = \frac{1}{5}$, then find the value of $\frac{abc}{ab+bc+ca}$?**
यदि $\frac{ab}{a+b} = \frac{1}{3}$, $\frac{bc}{b+c} = \frac{1}{4}$, $\frac{ca}{c+a} = \frac{1}{5}$ है, तो $\frac{abc}{ab+bc+ca}$ का मान ज्ञात कीजिये?

(a) $1/6$

(c) 6

$$\frac{1}{a} + \frac{1}{b} = 3$$

$$\frac{1}{b} + \frac{1}{c} = 4$$

$$\frac{1}{c} + \frac{1}{a} = 5$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 6$$

$$\frac{ab+bc+ca}{abc} = 6$$

(b) $1/12$

(d) $1/4$



वर्गसूत्र Square Formulae

$$a^2 - b^2 = (a + b)(a - b)$$

$$(a + b)^2$$

$$a^2 + b^2 + 2ab$$

$$(a - b)^2 + 4ab$$

$$(a - b)^2$$

$$a^2 + b^2 - 2ab$$

$$(a + b)^2 - 4ab$$

$$a^2 + b^2$$

$$(a + b)^2 - 2ab$$

$$(a - b)^2 + 2ab$$





$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$\# ab = \frac{(a + b)^2 - (a^2 + b^2)}{2}$$

$$a^4 + b^4 \Rightarrow (a^2 + b^2)^2 - 2a^2b^2$$

$$\frac{a + b}{a - b} + \frac{a - b}{a + b} = \frac{2(a^2 + b^2)}{(a^2 - b^2)}$$

$$\frac{a + b}{a - b} - \frac{a - b}{a + b} = \frac{4ab}{a^2 - b^2}$$

$$(a+b)^2 - (a-b)^2 = 4ab$$



घन सूत्र Cube Formulae

~~$(a - b)^3$~~

$(a - b)^3$ → $a^3 - b^3 - 3a^2b \oplus 3ab^2$

$(a - b)^3$ → $a^3 - b^3 - 3ab(a - b)$

$a^3 - b^3$ → $(a - b)^3 + 3ab(a - b)$

$a^3 - b^3$ → $(a - b)[(a - b)^2 + 3ab]$ ✓

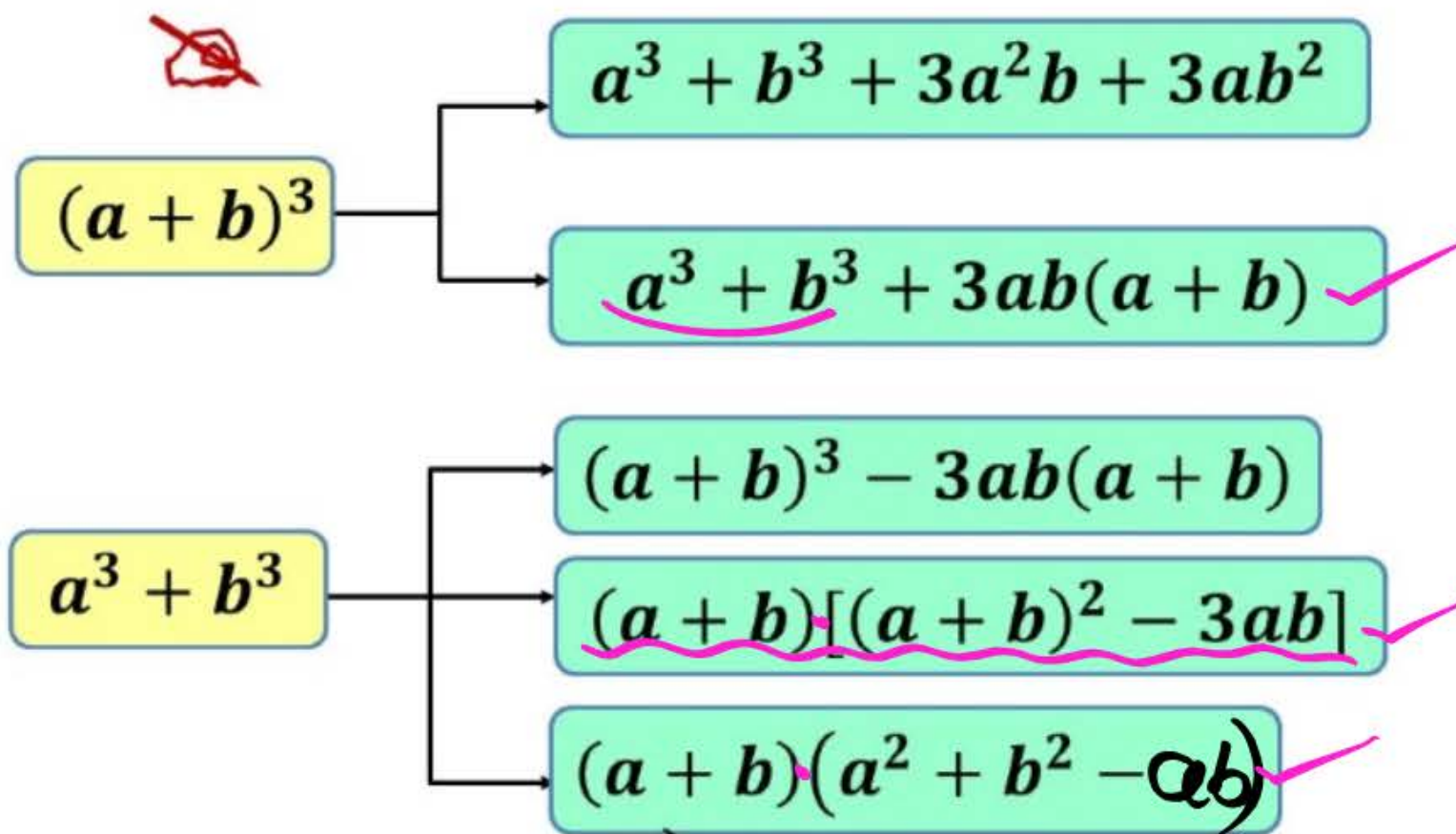
$a^3 - b^3$ → $(a - b)(a^2 + b^2 + ab)$ ✓

$$a^2 + b^2 + ab = \frac{a^3 - b^3}{a - b}$$

$$a - b = \frac{a^3 - b^3}{a^2 + b^2 + ab}$$



घन सूत्र Cube Formulae



$$a^2 + b^2 - ab = \frac{a^3 + b^3}{a + b}$$

$$a + b = \frac{a^3 + b^3}{a^2 + b^2 - ab}$$





1. If $x + y = 13$ and $x^2 + y^2 = 101$, then the value of $x^3 + y^3$ will be:

यदि $x + y = 13$ और $x^2 + y^2 = 101$, तो $x^3 + y^3$ का मान होगा:

(a) 767

(b) 949

(c) 793

(d) 871

$$= 13 \times (13^2 - 3 \times 34)$$

$$= 13 \times 67$$

$$\begin{aligned} xy &= \frac{13^2 - 101}{2} \\ &= 34 \end{aligned}$$

SelectionWay

2. If $x + y = 17$ and $xy = 25$, $x > y$, then the value of $(x^3 - y^3)$ is:

यदि $x + y = 17$ और $xy = 25$, $x > y$, है, तो $(x^3 - y^3)$ का मान है?

(a) $792\sqrt{21}$

(b) $702\sqrt{21}$

(c) $724\sqrt{23}$

(d) none

$$(x-y)^2 = (x+y)^2 - 4xy$$

$$= 289 - 100$$

$$= 189$$

$$x-y = \sqrt{189}$$

$$(x-y) [(x-y)^2 + 3xy]$$

$$= \sqrt{189} \times (189 + 75)$$

$$= \sqrt{189} \times 264$$

3. If $x + y = 4$ and $x^3 + y^3 = 12$, then the value of $x^4 + y^4$ will be:

यदि $x + y = 4$ और $x^3 + y^3 = 12$ है, तो $x^4 + y^4$ का मान होगा:

a) $\frac{146}{9}$ ✓

b) $\frac{131}{9}$

c) $\frac{143}{9}$

d) $\frac{127}{9}$

$$\cancel{12}^3 = \cancel{4} \cdot (4^2 - 3xy)$$

$$3xy = 13$$

$$xy = \frac{13}{3}$$

$$\begin{aligned} x^2 + y^2 &= (x+y)^2 - 2xy \\ &= 16 - \frac{26}{3} = \frac{22}{3} \end{aligned}$$

$$\begin{aligned} &= (x^2 + y^2)^2 - 2(xy)^2 \\ &= \frac{484}{9} - 2 \times \frac{169}{9} \\ &= \frac{146}{9} \end{aligned}$$

4. If $a^3 - b^3 = 260$, $a - b = 8$ then $a^4 + b^4 = ?$

a) 1682.5

b) 1628.5

c) 1862.5

d) 1268.5



5. If $25a^2 + 4b^2 = 124$, $ab = 3$ then $125a^3 - 8b^3 = ?$
a) 1322 b) 1264 c) 1232 d) 1268

$$(5a - 2b)^2 = 124 - 20 \times 3 = 64$$

$$5a - 2b = 8$$

$$\begin{aligned} & (5a)^3 - (2b)^3 \\ &= (5a - 2b) [(5a - 2b)^2 + 30ab] \\ &= 8 \times (64 + 90) \\ &= 1232 \end{aligned}$$

6. If $a^2 + b^2 = 69$ and $a^4 + b^4 = 3409$, then the value of $a^3 + b^3$ is :

यदि $a^2 + b^2 = 69$ और $a^4 + b^4 = 3409$ है, तो $a^3 + b^3$ का मान है:

(a) 506

(b) 407

(c) 429

(d) 473

$$a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$$

$$\begin{aligned} &= 11 \times (11^2 - 2 \times 26) \\ &= 11 \times 43 \end{aligned}$$

$$3409 = 4761 - 2a^2b^2$$

$$2a^2b^2 = 676$$

$$ab = 26$$

$$(a+b)^2 = 69 + 52 = 121$$

$$a+b = 11$$

7. If $x^3 = 116 + y^3$ and $x = 4 + y$, then find $(x + y)$?

A) $\frac{15}{\sqrt{6}}$

B) $\frac{10\sqrt{3}}{7}$

C) $10\sqrt{3}$

~~D) $\frac{10}{\sqrt{3}}$~~

$$x^3 - y^3 = 116$$

$$x - y = 4$$

$$\cancel{116} = \cancel{4} \cdot (4^2 + 3xy)$$

$$\frac{13}{3} = xy$$

$$= \sqrt{(x-y)^2 + 4xy}$$

$$= \sqrt{16 + \frac{52}{3}}$$

$$= \frac{10}{\sqrt{3}}$$

8. If $16x^2 + y^2 = \frac{65}{36}$ and $xy = \frac{1}{18}$, $x, y > 0$, then what is the value of $(64x^3 + y^3)$?

यदि $16x^2 + y^2 = \frac{65}{36}$ और $xy = \frac{1}{18}$, $x, y > 0$ है, तो $(64x^3 + y^3)$ का मान क्या होगा?

(a) $\frac{19}{4}$

(b) $\frac{27}{4}$

(c) $\frac{27}{8}$

(d) $\frac{19}{8}$

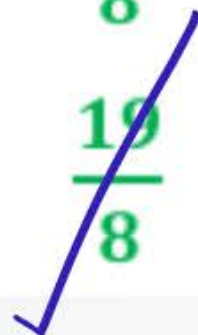
$$(4x + y)^2 = \frac{65}{36} + 8 \times \frac{1}{18} = \frac{81}{36}$$

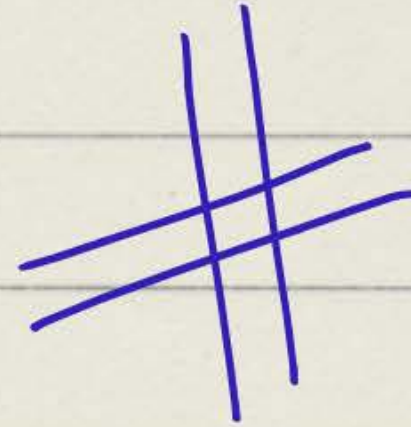
$$4x + y = \frac{9}{6} = \frac{3}{2}$$

$$(4x)^3 + y^3$$

$$= \frac{3}{2} \left(\frac{9}{4} \times \frac{1}{18} \right)$$

$$= \frac{3}{2} \times \frac{19}{4}$$





9. If $2s=a+b+c$ then find the value of $2s=a+b+c$

$$s(s-a)(s-b)(s-c) \left[\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s} \right]$$

यदि $2s=a+b+c$, तो $s(s-a)(s-b)(s-c) \left[\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s} \right]$ किसके बराबर है?

(UPSC CDS-1 2025)

- A) abc
- B) $2abc$
- C) $4abc$
- D) $ab+bc+ca$

10. If $pq+qr+rp=0$, then $\left(\frac{p^2}{p^2-qr} + \frac{q^2}{q^2-rp} + \frac{r^2}{r^2-pq}\right) = ?$

a) 3 b) 1 ✓ c) 2 d) 0

2 variable eqn
factor $\rightarrow p, q, r$

but $p=q=1$

$$1+r+r=0$$

$$r = -\frac{1}{2}$$

$$\frac{4}{6} + \frac{4}{6} + \frac{1}{-3}$$
$$= \frac{4}{3} - \frac{1}{3} = 1$$

$$p=1$$

$$q=1 \quad \text{या}$$

$$r = -\frac{1}{2}$$

2	✓
2	✓
-1	✓

11. If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$, then find $\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = ?$

a) 0 b) -2 c) 3 d) 1

1 eqⁿ
→ a, b & c

$$\text{put } c=0, b=1$$

$$a + \frac{1}{a} = 1 \Rightarrow a^3 = -1$$

$$a^2 + \frac{1}{a} = \frac{a^3 + 1}{a} = \frac{-1 + 1}{a}$$

SelectionWay

12. If $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 0$, then $\frac{ac}{b^2} + \frac{b^2}{ac} - \frac{c^3}{a^3} = ?$

a) -3 b) -2 c) -1 d) 0

put $b=c=1$

$$a + 1 + \frac{1}{a} = 0$$

$$\Rightarrow a + \frac{1}{a} = -1 \Rightarrow a^3 = -1$$

$$\begin{aligned} & \underline{a + \frac{1}{a} - \frac{1}{a^3}} \\ & \quad -1 -1 \\ & \quad = -2 \end{aligned}$$

13. If $a + b + \cancel{c} = 1$ and $a^3 + b^3 + \cancel{c^3} = 4$, then $\frac{1}{a+\cancel{b}} + \frac{1}{b+\cancel{a}} + \frac{1}{\cancel{c}ab} = ?$

a) 1 b) -1 **c) -2** d) 3

2 eqⁿ
1 eqⁿ for $a, b \& c$

put $c=0$

$$1 \times (1^3 - 3ab) = 4$$

$$-3ab = 3$$

$$ab = -1$$

$$= \frac{1}{a} + \frac{1}{b} + \frac{1}{ab}$$

$$= \frac{a+b+1}{ab}$$

$$= \frac{2}{ab}$$

$$= -2$$

SelectionWay

14. If $\left[\sqrt{a^2 + b^2 + ab}\right] + \left[\sqrt{a^2 + b^2 - ab}\right] = 1$, then what is the value of $(1-a^2)(1-b^2)$?

यदि $\left[\sqrt{a^2 + b^2 + ab}\right] + \left[\sqrt{a^2 + b^2 - ab}\right] = 1$ है, तो $(1-a^2)(1-b^2)$ का मान क्या है?

(a) $1/4$

(c) $5/4$

(b) $4/7$

✓ (d) $3/4$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

put $b=0$

$$2a=1$$

$$a=\frac{1}{2}$$

15. What is $\frac{x^2 - y^2 - z^2 - 2yz}{x^2 + y^2 - z^2 + 2xy} + \frac{x^2 - y^2 - z^2 - 2yz}{x^2 - y^2 + z^2 - 2xz}$ equal to?

$\frac{x^2 - y^2 - z^2 - 2yz}{x^2 + y^2 - z^2 + 2xy} + \frac{x^2 - y^2 - z^2 - 2yz}{x^2 - y^2 + z^2 - 2xz}$ किसके बराबर है? (CDS 2023)

A) $\frac{x}{x+y-z} = 1$

B) $\frac{y+z}{x+y-z}$

C) $\frac{2x}{x+y-z} = 2$

D) $\frac{2y+2z}{x+y-z}$

but $z=0, y=0$

$= 2$