

27. If x is the remainder when 3^{61284} is divided by 5 and y is the remainder when 4^{96} is divided by 6, then what is the value of $(2x-y)$?

यदि 3^{61284} को 5 से विभाजित किया जाता है तो शेषफल x बचता है और यदि 4^{96} को 6 से विभाजित किया जाता है तो शेषफल y बचता है। $(2x-y)$ का मान क्या है?

(a) 4

(b) -2

(c) 2

(d) -4

$$\frac{3^{\textcircled{61284}}}{5} = x = 1 \quad \text{Rem}$$

$$\frac{3^{\textcircled{4}}}{5} = 1 \quad \text{Rem}$$

$$\frac{4^{96}}{6} = 4 = y \quad \text{Rem}$$

$$2 \times 1 - 4 = -2$$

special case

$$\frac{4^1}{6} \stackrel{\text{(rem)}}{=} 4$$

$$\frac{4^2}{6} = 4$$

$$\frac{4^3}{6} = 4$$

$$\frac{4^4}{6} = 4$$

$$\frac{4^n}{6} \stackrel{\text{(rem)}}{=} 4$$

$$\frac{\cancel{10}^{37}}{6} = \frac{4^{37}}{6} \stackrel{\text{(rem)}}{=} 4$$



Euler's theorem

$$\overset{\text{Rem}}{a^{\Phi(N)}} \underset{N}{=} 1 \checkmark$$

N = any natural no

$\Phi(N)$ = Totient function of 'N'

$a, N \in \text{coprime}$

$$N = 80 = 2^4 \times 5^1$$

$$\Phi(80) = \cancel{80}^8 \times \frac{1}{2} \times \frac{4}{5} = 32$$

$$N = 12 = 2^2 \times 3^1$$

$$\Phi(N) = \cancel{12}^2 \times \frac{1}{2} \times \frac{2}{3} = 4$$

12 → (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)

Co - prime → (12, 1), (12, 5), (12, 7), (12, 11)

12 से छोटी 4 संख्याएँ 12 के साथ co-prime हैं।

$\Phi(N)$ = N से छोटी कितनी संख्याएँ N के साथ co-prime हैं।

$$N = 75 = 5^2 \times 3^1$$

$$\Phi(75) = \cancel{75}^5 \times \frac{4}{5} \times \frac{2}{3} = 40$$

$$N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$$

$$\Phi(N) = \cancel{420}^2 \times \frac{1}{2} \times \frac{2}{3} \times \frac{6}{7} \times \frac{4}{5} = 96$$

$$\frac{43^{32}}{80} = \text{(Rem)} 1$$

$$\Phi(80) = 32$$

$$\frac{85^{72}}{54} = \frac{(85^{18})^4}{54} = 1^4 = 1 \text{ Rem}$$

$$\Phi(54) = 18$$

$$\frac{85^{18}}{54} = 1 \text{ Rem}$$

$$\frac{59^{82}}{48} = \frac{(59^6)^5 \times 59 \times 59}{48} = 1 \times 59 \times 59 = 1 \text{ Rem}$$

$$\Phi(48) = 16$$

$$\frac{59^{16}}{48} = 1 \text{ Rem}$$





$$7 = \textcircled{7}'$$

$$\Phi(7) = \cancel{7} \times \frac{6}{\cancel{7}} = 6$$



$$\frac{a^{p-1}}{p} \text{ (Rem)} = 1$$

where p is any prime no

Fermat's Theorem

$$\Phi(7) = 7 \times \frac{6}{7} = 6$$

$$\Phi(P) = (P - 1)$$

(p = any prime no)

$$\frac{a^{\Phi(N)}}{N} = 1$$

28. Find the remainder: (Euler's Remainder Theorem)

शेषफल ज्ञात कीजिए: (यूलर का शेषफल प्रमेय)

(a) $\frac{13^{127}}{49}$

(c) $\frac{53^{98}}{90} \rightarrow 19$

(e) $\frac{32^{32^{32}}}{7}$

(b) $\frac{5^{75}}{18} = \frac{(5^6)^{12} \times 5^3}{18} = 17$

(d) $\frac{23^{722} + 24^{722}}{25} \rightarrow \frac{23^{722}}{25} + \frac{24^{722}}{25}$

(f) $\frac{35^{33^{31}}}{16}$

$\phi(25) = 25 \times \frac{4}{5} = 20$

$\frac{23^{20}}{25}$ Answer

$= \frac{23^2}{25} + 1$
 $= 4 + 1 = 5$ ✓



$$\frac{\cancel{32}^{32}}{7} = \frac{4^{\cancel{32}}}{7} = \frac{4^4}{7} = \frac{16 \times 16}{7} = 4^{\text{Rem}}$$

$$\frac{4^6}{7} = 1^{\text{Rem}}$$

$$\frac{\cancel{32}^{32}}{6} = \frac{2^{32}}{6} = \frac{4^{16}}{6} = 4^{\text{Rem}}$$

$$(2^{32})^6$$



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$$\frac{35\textcircled{7}}{10} = \overset{\checkmark}{7} \text{ (Rem)}$$

$$\frac{54\textcircled{89}}{100} = 89 \text{ (Rem)}$$

$$\frac{7386\textcircled{127}}{1000} = 127 \text{ (Rem)}$$



29. Find the last 2-digits of: 43^{362}

43^{362} के अंतिम 2 अंक ज्ञात कीजिए

$$\frac{43^{\cancel{362}}}{100} = \frac{43^2}{100} = \frac{1849}{100} = 49$$

$$2^2 \times 5^2$$
$$\phi(100) = 100 \times \frac{1}{2} \times \frac{4}{5} = 40$$

$$\frac{43^{40}}{100} = 1^{\text{Rem}}$$

30. Find the remainder: $\frac{2^{100}}{102}$

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शेषफल ज्ञात कीजिए: $\frac{2^{100}}{102} = \frac{\cancel{2} \times \cancel{2}^{99}}{\cancel{102}} = \frac{\cancel{2}^{99}}{51} = \frac{2^3}{51} = 8$

$\div 2$

17×3

$$8 \times 2 = 16 \text{ Rem } \checkmark$$

$$\phi(51) = 5 \times \frac{16}{17} \times \frac{2}{3} = 32$$

$$\frac{2^{32}}{51} = 1 \text{ Rem}$$

31. Find the remainder: / शेषफल ज्ञात कीजिए:

$$\frac{10^1 + 10^2 + 10^3 + 10^4 + \dots + 10^{100}}{6} = ?$$

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(CDS)

$$\frac{10^n}{6} = \frac{4^n}{6} = 4^{\text{Rem}}$$

$$= 4 \times 100$$

$$= \frac{400}{6} = \frac{\text{Rem}}{4}$$

SelectionWay

32.

Find the remainder: / शेषफल

$$\frac{1^{93} + 2^{93} + 3^{93} + \dots + 88^{93} + 89^{93}}{90} = ?$$

$$\frac{45^{93}}{90} =$$

$$\frac{45^{92}}{2} = \text{Rem } 45$$

$$\div 45 \checkmark$$

33. What is the remainder when $7+77+777+7777+\dots+(\text{till } 100 \text{ terms})$ is divided by 8?

$7+77+777+7777+\dots+(100 \text{ पदों तक})$ को 8 से विभाजित करने पर क्या शेषफल है ?

(a) 6

(b) 0

(c) 1

(d) 7

$$\textcircled{7} + \textcircled{5} + \textcircled{1} + \textcircled{1} \dots \dots \dots 98 \text{th term}$$

$$\begin{array}{r} 7+77+777+7777+\dots+100 \text{ terms} \\ \hline 8 \end{array}$$

$$12+98 = \frac{110}{8} = 6 \text{ Rem}$$

$$+ 9 = 6 \times 3$$

$$+ 5 = 1 + 4$$

$$666 = 6 \times 3 \times 37$$

$$abcabc = abc \times 1001$$

$$\dot{a}\dot{a}\dot{a} = a \times \underline{111}$$

$$aaa = a \times 3 \times 37$$

$$\overleftrightarrow{aaaaaa} = aaa \times 1001$$

$$aaaaaa = 8 \times 3 \times 37 \times 7 \times 11 \times 13$$

$$\frac{888888}{39} = \text{Rem } 0$$





$$\begin{array}{r} 55555555 \\ \hline 37 \end{array} = 5 \text{ (Rem)}$$

34. What is the remainder when 7272727272.....till 999 digit is divided by 101?

7272727272.....999 अंक तक को 101 से विभाजित करने पर क्या शेषफल है ?

(a) 0

(b) 20

(c) 27

(d) 7

$$\overline{7272727272 \dots 999 \text{ digits}} = \frac{727}{101} = 20 \text{ Rem}$$

$$abab = ab \times 101$$

$$996 \div 3$$



35. A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?

9 को 99 बार लिखने पर एक संख्या N बनती है। यदि N को 13 से विभाजित किया जाए तो शेषफल क्या होगा?

(a) 11

(b) 9

(c) 7

(d) 1

$$N = \underbrace{999999}_{13} \dots 99 \text{ बार} = \frac{999}{13} = 11 \text{ Rem}$$

$$99 \rightarrow 6 \times 16 + 3$$