

$$\text{If, } x + \frac{1}{x} = \pm\sqrt{2},$$

$$x^2 + \frac{1}{x^2} = 0$$

$$x^4 + 1 = 0$$

$$x^4 = -1 \checkmark$$



If $x + \frac{1}{x} = \pm\sqrt{3}$ or $x^2 + \frac{1}{x^2} = 1$,

then

$$x^3 + \frac{1}{x^3} = 0$$

$$x^6 + 1 = 0 \longrightarrow$$

$$x^6 = -1$$



Q30. If $\left(x + \frac{1}{x} + 1\right)\left(x + \frac{1}{x} - 1\right) = 2$, then

यदि $\left(x + \frac{1}{x} + 1\right)\left(x + \frac{1}{x} - 1\right) = 2$, तो

(i) $x^{38} + \frac{1}{x^{38}} = ?$

(ii) $x^{53} + \frac{1}{x^{53}} = ?$

(iii) $x^{320} + x^{314} + x^{250} + x^{244} + x^{94} + x^{88} + x^{45} + x^{39} + x^6 + 5 = ?$
 $= -1 + 5 = 4$

$$\left(x + \frac{1}{x}\right)^2 - 1^2 = 2$$

$$x + \frac{1}{x} = \sqrt{3} \Rightarrow x^6 + 1 = 0$$

$$x^6 = -1$$

$$x^{53} = \frac{x^{54}}{x} = \frac{(x^6)^9}{x} = \frac{-1}{x}$$



If $x + \frac{1}{x} = K$, then

$$x^5 \oplus \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right) \cdot \left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

If $x - \frac{1}{x} = K$, then

$$x^5 \ominus \frac{1}{x^5} = \left(x^3 - \frac{1}{x^3}\right) \cdot \left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right)$$



31. If $x > 0$, and $x^4 + \frac{1}{x^4} = 254$, what is the value of $x^5 + \frac{1}{x^5}$?

यदि $x > 0$, और $x^4 + \frac{1}{x^4} = 254$, तो $x^5 + \frac{1}{x^5}$ का मान क्या है?

(a) $717\sqrt{2}$

(c) $720\sqrt{2}$

(b) $723\sqrt{2}$

(d) $726\sqrt{2}$

$$x^2 + \frac{1}{x^2} = 16$$

$$x + \frac{1}{x} = \sqrt{18}$$

$$15\sqrt{18} \times 16 - \sqrt{18}$$

$$= 239\sqrt{18}$$

$$= 239 \times 3\sqrt{2}$$



32. If $X = 4 - \frac{1}{4 - \frac{1}{4 - \frac{1}{4 - \dots \dots \dots \infty}}}$, then find $X^5 + \frac{1}{X^5} = ?$

यदि $X = 4 - \frac{1}{4 - \frac{1}{4 - \frac{1}{4 - \dots \dots \dots \infty}}}$, तो $X^5 + \frac{1}{X^5} =$ ज्ञात करें?

(a) 656

(c) 732

(b) 548

(d) 724

$$x = 4 - \frac{1}{x}$$

$$x + \frac{1}{x} = 4$$



33. If $x - \frac{1}{x} = -6$, what will be the value of $x^5 - \frac{1}{x^5}$?

यदि $x - \frac{1}{x} = -6$ है, तो $x^5 - \frac{1}{x^5}$ का मान क्या होगा?

SSC CGL 2023 PRE

(a) ~~-8898~~
(c) -8886

(b) ~~-8896~~
(d) ~~-8892~~

$$\begin{aligned} & \left(x^3 - \frac{1}{x^3}\right) \left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) \\ & \quad \downarrow \quad \quad \downarrow \\ & -234x \quad 38+6 \\ & = -\underline{(234 \times 38 - 6)} = \end{aligned}$$



If $x \oplus \frac{1}{x} = K$, then $x^6 \oplus \frac{1}{x^6} = ?$

$$x^3 + \frac{1}{x^3} = K^3 - 3K = T$$

$$x^6 + \frac{1}{x^6} = T^2 - 2$$



If $x - \frac{1}{x} = K$, then $x^6 + \frac{1}{x^6} = ?$

$$x^3 - \frac{1}{x^3} = K^3 + 3K = T$$

$$x^6 \textcircled{+} \frac{1}{x^6} = T^2 + 2$$



34. If $\left(a + \frac{1}{a}\right) = 7\sqrt{3}$, then what is the value of $(a^6 + a^{-6})$?

यदि $\left(a + \frac{1}{a}\right) = 7\sqrt{3}$ है, तो $(a^6 + a^{-6})$ का मान ज्ञात कीजिए।

(a) 3048190

(b) 3048542 $\rightarrow 145 - 3 \times 145$

(c) 3048132

(d) 3048625

$$a^2 + \frac{1}{a^2} = (7\sqrt{3})^2 - 2 = 145$$



35. If $\left(x - \frac{1}{x} = 4\right)$, then what is the value of $\left(x^6 + \frac{1}{x^6}\right)$?

यदि $\left(x - \frac{1}{x} = 4\right)$, तो $\left(x^6 + \frac{1}{x^6}\right)$ का मान क्या है?

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(a) 4689

(b) 4786

(c) 5832

(d) 5778 ✓

$$\begin{aligned} x^3 - \frac{1}{x^3} &= 4^3 + 3 \times 4 \\ &= 76 \end{aligned}$$

$$76^2 + 2$$



if, $x + \frac{1}{x} = 6$ & $x > 1$

$$x - \frac{1}{x} = \sqrt{32}$$

तो $x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right)$

$\downarrow \qquad \downarrow$
 $198 \times 35\sqrt{2} \checkmark$

$$= 6930\sqrt{2}$$

$$= 27720\sqrt{2}$$

If $x \oplus \frac{1}{x} = K$, then

$$x^7 + \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

~~Wrong~~

If $x \ominus \frac{1}{x} = K$, then

$$x^7 \ominus \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right) \left(x^3 - \frac{1}{x^3}\right) \oplus \left(x - \frac{1}{x}\right)$$



36. If $x > 0$ & $x^4 + \frac{1}{x^4} = 2207$, then find $x^7 + \frac{1}{x^7}$?

यदि $x > 0$ & $x^4 + \frac{1}{x^4} = 2207$, तो $x^7 + \frac{1}{x^7}$ का मान क्या है?

(a) 710649

(c) 710654

(b) 710647

(d) 710661

$$x^4 + \frac{1}{x^4} = 2207$$

$$x + \frac{1}{x} = 7$$

$$2207 \times 3 - 7 = 710647$$



37. If $x + \frac{1}{x} = 4$, then, $x^7 - \frac{1}{x^7} = ?$

यदि $x + \frac{1}{x} = 4$, तो, $x^7 - \frac{1}{x^7} = ?$

a) $5844\sqrt{3}$
 c) $5824\sqrt{3}$

b) $5842\sqrt{3}$
 ✓ d) $5822\sqrt{3}$

$$x - \frac{1}{x} = \sqrt{4^2 - 4} = \sqrt{12}$$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^3 - \frac{1}{x^3}\right) + \left(x - \frac{1}{x}\right)$$

$$= 194 \times 15\sqrt{12} + \sqrt{12}$$

$$= 2911\sqrt{12}$$

$$= 5822\sqrt{3}$$



38. If $\left(x - \frac{1}{x}\right) = \sqrt{6}$ & $x > 1$, then what is the value of $\left(x^8 - \frac{1}{x^8}\right)$?

यदि $\left(x - \frac{1}{x}\right) = \sqrt{6}$ & $x > 1$, तो $\left(x^8 - \frac{1}{x^8}\right)$ का मान क्या है?

(a) $1024\sqrt{15}$

☒ (b) $992\sqrt{15}$

(c) $998\sqrt{15}$

(d) $1012\sqrt{15}$

$$\left(x^4 + \frac{1}{x^4}\right) \cdot \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) \cdot \left(x - \frac{1}{x}\right)$$

$$\begin{array}{cccc} \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 6x & \times & 8 & \times & \sqrt{10} & \times & \sqrt{6} \end{array}$$

$$= 496 \times 2\sqrt{15}$$



39. $x^2 - 3x + 1 = 0$, then find the value of $x^9 + x^7 + x^{-9} + x^{-7}$?

$x^2 - 3x + 1 = 0$, तब $x^9 + x^7 + x^{-9} + x^{-7}$? का मान बताओ

a) 6621

b) 4414

c) 9208

d) 6069

$x^2 + 1 = 3x$

$x + \frac{1}{x} = 3$

$7 \times 3 - 2 = 19$

$= x^7(x^2 + 1) + \frac{(1 + x^2)}{x^9}$

$= 3x^8 + \frac{3}{x^8}$

$= 3(x^8 + \frac{1}{x^8})$

$= 3 \times 19 = 57$



40. If $x^2 - 3x + 1 = 0$, then what is the value of $x^{12} + x^8 - 123x^7 - 123x^3$?

यदि $x^2 - 3x + 1 = 0$, तो $x^{12} + x^8 - 123x^7 - 123x^3$ का मान क्या है?

(a) 7

(b) -3

(c) -7

(d) 3

#

$$x + \frac{1}{x} = 3$$

$$x^5 + \frac{1}{x^5} = 18x^7 - 3$$

$= 123$

$$= x^{12} + x^8 - \left(x^5 + \frac{1}{x^5}\right)x^7 - \left(x^5 + \frac{1}{x^5}\right)x^3$$

$$= \cancel{x^{12}} + \cancel{x^8} - \cancel{x^{12}} - \cancel{x^8} - \cancel{x^8} - \frac{1}{\cancel{x^2}}$$
$$= -\left(x^2 + \frac{1}{x^2}\right)$$
$$= -7$$



41. If $x^3 - \frac{1}{x^3} = \sqrt{k^2 - 4}(k^2 - 1)$, then $x^2 - \frac{1}{x^2} = ?$

यदि $x^3 - \frac{1}{x^3} = \sqrt{k^2 - 4}(k^2 - 1)$, तो $x^2 - \frac{1}{x^2} = ?$

(a) $k\sqrt{k^2 - 4}$
(c) $k(k^2 - 4)$

(b) $k\sqrt{k^2 + 4}$
(d) $k(k^2 + 4)$

if, $x - \frac{1}{x} = \sqrt{x}$

$x^3 - \frac{1}{x^3} = \sqrt{x} \cdot (x+3)$

$$\begin{cases} x - \frac{1}{x} = \sqrt{k^2 - 4} \\ x + \frac{1}{x} = k \end{cases}$$

42. If $x - \frac{1}{x} = 1$, then $\left\{ \frac{1}{x-1} - \frac{1}{x+1} + \frac{1}{x^2+1} - \frac{1}{x^2-1} \right\} = ?$

यदि $x - \frac{1}{x} = 1$, तो $\left\{ \frac{1}{x-1} - \frac{1}{x+1} + \frac{1}{x^2+1} - \frac{1}{x^2-1} \right\} = ?$

(a) $\pm\sqrt{5}$

(b) $\frac{2}{5}$

(c) $\pm \frac{2}{\sqrt{5}}$

(d) $\pm \frac{\sqrt{5}}{2}$

$$\begin{aligned} & \frac{2}{\left(x - \frac{1}{x}\right) \cdot \left(x + \frac{1}{x}\right)} \\ &= \frac{2}{1x \pm \sqrt{5}} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{x^2-1} - \frac{1}{x^2+1} \\ &= \frac{x^2/x^2}{(x^2-1) \cdot (x^2+1)} \end{aligned}$$

43. **If $x\sqrt{x} + \frac{1}{x\sqrt{x}} = 110$, then find $x^2\sqrt{x} + \frac{1}{x^2\sqrt{x}}$?**

यदि $x\sqrt{x} + \frac{1}{x\sqrt{x}} = 110$ है, तो $x^2\sqrt{x} + \frac{1}{x^2\sqrt{x}}$ ज्ञात कीजिये?

(a) 2525

(b) 12098

(c) 140

(d) 3645

$$\sqrt{x}^3 + \frac{1}{\sqrt{x}^3} = 110$$

$$\sqrt{x}^5 + \frac{1}{\sqrt{x}^5} = 2525$$

$$\sqrt{x} + \frac{1}{\sqrt{x}} = 5$$



If $x + \frac{1}{x} = 2$, then $x = 1$

& If $x + \frac{1}{x} = -2$, then $x = -1$



44. If $x + \frac{1}{x-9} = 11$, then $(x-11)^{12} + \frac{1}{(x-11)^{13}} = ?$

यदि $(x-9) + \frac{1}{(x-9)} = 11$, तो $(x-11)^{12} + \frac{1}{(x-11)^{13}} = ?$

(a) 0

|—|

(b) 2

(c) -2

= 0

(d) 1

$$(x-9) + \frac{1}{(x-9)} = 11$$

$$x = 10$$



If $x + \frac{1}{x} = 1$ or $x^2 - x + 1 = 0$,

then

$$x^3 = -1$$

#



If $x + \frac{1}{x} = -1$ or $x^2 + x + 1 = 0$,

then

$$x^3 = 1$$



45. If, $\frac{r}{13} + \frac{13}{r} = 1$ then the value of r^3 is:

यदि $\frac{r}{13} + \frac{13}{r} = 1$ है, तो r^3 का मान ज्ञात कीजिए।

(SSC CGL 2022)

(a) -2157

(b) 2197

(c) 2157

✓ (d) -2197

$$\left(\frac{-13}{13}\right)^3 = -1$$



46. If $x + \frac{1}{x} = 1$, then $(1+x+x^2)(1-x-x^2)=?$

यदि $x + \frac{1}{x} = 1$, तो $(1+x+x^2) \cdot (1-x-x^2)=?$

(a) 0

(b) 4

(c) -2

(d) 1

$$x^2 + 1 = x$$
$$1 - x = -x^2$$

$$\Rightarrow x^3 = -1$$

$$= x^2 x - x^2$$

$$= -4x^3$$

$$= -4(-1)$$
$$= 4$$



47. If $\frac{x}{5\sqrt{6}} + 1 = \frac{-5\sqrt{6}}{x}$, then find the value of $x^5 - 750\sqrt{6}x^2 + 1 = ?$

यदि $\frac{x}{5\sqrt{6}} + 1 = \frac{-5\sqrt{6}}{x}$, तो $x^5 - 750\sqrt{6}x^2 + 1$ का मान ज्ञात कीजिए?

(a) $25\sqrt{6}$

✓ (b) 1

(c) 0

(d) $1+5\sqrt{6}$

$$\left(\frac{x}{5\sqrt{6}}\right) + \left(\frac{5\sqrt{6}}{x}\right) = -1$$

$$\Rightarrow \left(\frac{x}{5\sqrt{6}}\right)^3 = 1$$

$$x^3 = 750\sqrt{6}$$

48. If $x^2 - 7x + 13 = 0$ then $(x - 3)^{22} + \frac{1}{(x-3)^{22}} = ?$

यदि $x^2 - 7x + 13 = 0$ तो $(x - 3)^{22} + \frac{1}{(x-3)^{22}} = ?$

(a) 3

(c) -1

(b) 1

(d) -2

$$y^2 + 6y + 9 - 7y - 21 + 13 = 0$$

$$\frac{y^2 - y + 1}{y} = 0$$

$$y + \frac{1}{y} = 1 \Rightarrow y^3 = -1$$

$$y^{22} = (y^3)^7 \times y = -y$$

माना $x - 3 = y$

$$\Rightarrow x = y + 3$$

$$y^{22} + \frac{1}{y^{22}} = -y - \frac{1}{y} = -1$$

