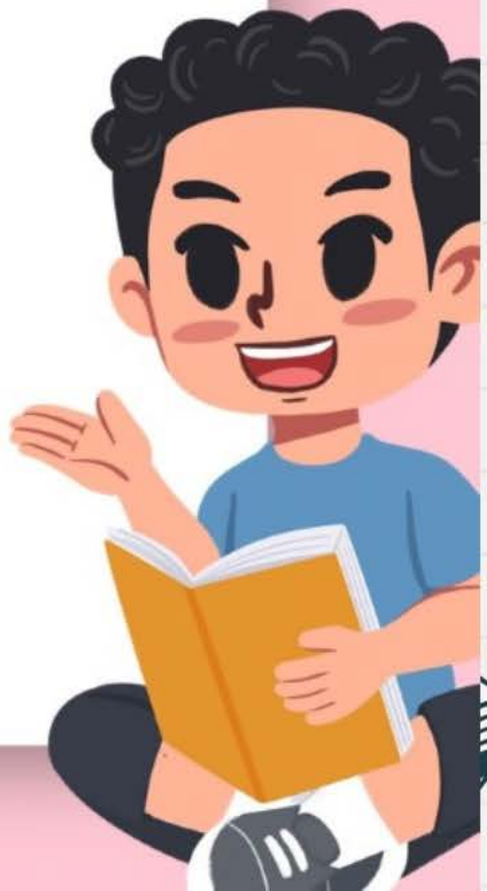
IMPORTANT ALGEBRIC IDENTITIES

If $x + \frac{1}{x} = K$,

then $x^2 + \frac{1}{x^2} = K^2 - 2 = T$

Now, $x^4 + \frac{1}{x^4} = T^2 - 2 = U$

& $x^8 + \frac{1}{x^8} = U^2 - 2$



SelectionWay
Test RanKING

If $x - \frac{1}{x} = K$,

then $x^2 + \frac{1}{x^2} = K^2 + 2 = T$

Now, $x^4 + \frac{1}{x^4} = T^2 - 2 = U$

& $x^8 + \frac{1}{x^8} = U^2 - 2$

SelectionWay
Test RanKING

If $Ax + \frac{B}{Cx} = K$,

then $A^2x^2 + \frac{B^2}{C^2x^2} = K^2 - 2 \times A \times \frac{B}{C}$

If $Ax - \frac{B}{Cx} = K$,

then $A^2x^2 + \frac{B^2}{C^2x^2} = K^2 + 2 \times A \times \frac{B}{C}$

1. If $x = 10 + 3\sqrt{11}$, then find $\sqrt{x} + \frac{1}{\sqrt{x}}$?

यदि $x = 10 + 3\sqrt{11}$ है, तो $\sqrt{x} + \frac{1}{\sqrt{x}}$ ज्ञात कीजिये?

$\frac{1}{x} = 10 - 3\sqrt{11}$

(a) 4.84 (b) 5.69

(c) 4.69 (d) 3.92

$x + \frac{1}{x} = 20$

$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2$

$= 22$

$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{22}$ ✓

$\approx$

$4.8^2 = 23.04$

2. If $x + \frac{49}{x+48} = -34$, then find $(2x + 83)^3 + \frac{1}{(2x+83)^3}$?

यदि $x + \frac{49}{x+48} = -34$ है, तो $(2x + 83)^3 + \frac{1}{(2x+83)^3}$ ज्ञात कीजिये?

(a) 7 (b) 2 ✓ 1+1

(c) -2 (d) 1

$$(x+48) + \frac{49}{(x+48)} = 14$$

$x+48 = 7$

$x = -41$

$a + \frac{49}{a} = 14$

$\Rightarrow a = 7$

3. If $a = \frac{\sqrt{5}+2}{\sqrt{5}-2}$ and $b = \frac{\sqrt{5}-2}{\sqrt{5}+2}$, then the value of $2a^2 + 2b^2 - 5ab$ is equal to:

यदि $a = \frac{\sqrt{5}+2}{\sqrt{5}-2}$ और $b = \frac{\sqrt{5}-2}{\sqrt{5}+2}$ है, तो $2a^2 + 2b^2 - 5ab$ का मान ज्ञात कीजिए।

(a) 693 (b) 649

(c) 635 (d) 639

$a \cdot b = 1$

$b = \frac{1}{a}$

$= 2\left(a^2 + \frac{1}{a^2}\right) - 5$

$= 2 \times 329 - 5$

$a + \frac{1}{a} = \frac{2 \times 9}{1} = 18$

4. If $a2 - 13a - 1 = 0$, then find the value of $a^2 + \frac{1}{a^2}$.

यदि $a^2 - 13a - 1 = 0$ हो, तो $a^2 + \frac{1}{a^2}$ का मान ज्ञात कीजिए।

[RRB Group D-2022]

[A] 13
[B] 169
[C] 167
[D] 171

$a - \frac{1}{a} = 13$

$13^2 + 2$

5. If $\sqrt{x} - \frac{1}{\sqrt{x}} = 7$, then $x^2 + \frac{1}{x^2}$ is equal to:

यदि $\sqrt{x} - \frac{1}{\sqrt{x}} = 7$, हो, तो $x^2 + \frac{1}{x^2}$ का मान होगा?

(a) 2599 (b) 2603

(c) 2508 (d) 2560

$= 51^2 - 2$

$x + \frac{1}{x} = 7^2 + 2 = 51$

If $x + \frac{1}{x} = K$, then

$$x - K = -\frac{1}{x}$$

$$x(x - K) = -1$$

or

$$x^2 - Kx + 1 = 0$$

If $x - \frac{1}{x} = K$, then

$$x - K = \frac{1}{x}$$

$$x(x - K) = 1$$

or

$$x^2 - Kx - 1 = 0$$

6. If $x^2 - 11x + 1 = 0$, then find $x^8 - 14159x^4 + 11$?

यदि $x^2 - 11x + 1 = 0$, तो $x^8 - 14159x^4 + 11$ ज्ञात करें?

(a) 9 (b) 10

(c) 12 (d) 11

$x + \frac{1}{x} = 11$

$11^2 - 2 = 119$

$x^4 + \frac{1}{x^4} = 14159$

$x^4(x^4 - 14159) + 11$

$-1 + 11$

7. If $12x^2 - 21x + 1 = 0$, then what is the value of $9x^2 + (16x^2)^{-1}$?

यदि $12x^2 - 21x + 1 = 0$, तो $9x^2 + (16x^2)^{-1}$ का मान क्या है?

(a) $\frac{429}{8}$

(b) $\frac{465}{16}$

(c) $\frac{417}{16}$

(d) $\frac{453}{8}$

$4x$

$3x + \frac{1}{4x} = \frac{21}{4}$

$3x + \frac{1}{4x}$ ✓

$9x^2 + \frac{1}{16x^2} = \frac{441}{16} - 2 \times 3 \times \frac{1}{4}$

$= \frac{441 - 24}{16}$

8. If $x^4 + \frac{1}{x^4} = 6887$, then the positive value of $x - \frac{1}{x}$ is:

यदि $x^4 + \frac{1}{x^4} = 6887$ है, तो $x - \frac{1}{x}$ का धनात्मक मान ज्ञात करें।

(a) 12 (b) 9

(c) 15 (d) 8

#

$$x^2 + \frac{1}{x^2} = \sqrt{6889} = 83$$

$$x - \frac{1}{x} = \sqrt{83 - 2} = 9$$

$$x + \frac{1}{x} = \sqrt{83 + 2} = \sqrt{85}$$

9. If $x^4 + \frac{16}{x^4} = 27217, x > 0$, then the value of $x + \frac{2}{x}$ is :

यदि $x^4 + \frac{16}{x^4} = 27217, x > 0$ है, तो $x + \frac{2}{x}$ का मान क्या होगा?

(a) 17 (b) 11

(c) 15 (d) 13

$$\left(x^2\right)^2 + \left(\frac{4}{x^2}\right)^2 + 8 = 27225$$

$$x^2 + \frac{4}{x^2} + 4 = 165 + 4$$

$$x + \frac{2}{x} = 13$$

10. If $x^4 + x^{-4} = 119, (x > 0)$, then the value of $(2x - 3)^2$ is:

यदि $x^4 + x^{-4} = 119$ है, तो $(2x - 3)^2$ का मान है:

(a) 12 (b) 13

(c) 15 (d) 14

$$x^4 + \frac{1}{x^4} = 119$$

$$x^2 + \frac{1}{x^2} = \sqrt{121} = 11$$

$$x - \frac{1}{x} = 3 \Rightarrow x(x-3) = 1$$

$$4x^2 - 12x + 9$$

$$= 4x(x-3) + 9$$

$$= 4 \times 1 + 9$$

If $x + \frac{1}{x} = K$, then $x - \frac{1}{x} = \pm\sqrt{K^2 - 4}$

If $x - \frac{1}{x} = K$, then $x + \frac{1}{x} = \pm\sqrt{K^2 + 4}$

If $Ax + \frac{B}{Cx} = K$, then

$$Ax - \frac{B}{Cx} = \pm\sqrt{K^2 - 4 \times A \times \frac{B}{C}}$$

If $Ax - \frac{B}{Cx} = K$, then

$$Ax + \frac{B}{Cx} = \pm\sqrt{K^2 + 4 \times A \times \frac{B}{C}}$$

11. If $\frac{m^{117}}{n^{109}} - \frac{n^{109}}{m^{117}} = 9$, then find $\frac{m^{117}}{n^{109}} + \frac{n^{109}}{m^{117}}$?

यदि $\frac{m^{117}}{n^{109}} - \frac{n^{109}}{m^{117}} = 9$ है, तो $\frac{m^{117}}{n^{109}} + \frac{n^{109}}{m^{117}}$ ज्ञात कीजिये?

(a) $\sqrt{90}$

(b) $\sqrt{85}$

(c) $\sqrt{10}$

(d) $\sqrt{82}$

$x - \frac{1}{x} = 9$

$x + \frac{1}{x} = \sqrt{9^2+4}$

12. If $\frac{A+B}{\sqrt{AB}} = 10\sqrt{2}$, then $\frac{A-B}{\sqrt{AB}} = ?$

यदि $\frac{A+B}{\sqrt{AB}} = 10\sqrt{2}$, तो $\frac{A-B}{\sqrt{AB}} = ?$

(a) 12

(b) 14

(c) 13

(d) $9\sqrt{2}$

$\sqrt{\frac{A}{B}} + \sqrt{\frac{B}{A}} = 10\sqrt{2}$

$\sqrt{200-4}$

13. If $x + \frac{1}{x+7} = 0$, then $x - \frac{1}{x+7} = ?$

यदि $x + \frac{1}{x+7} = 0$, हो तो, $x - \frac{1}{x+7} = ?$

(a) $3\sqrt{5} - 5$ (b) $3\sqrt{5} - 7$

(c) $3\sqrt{7} - 5$ (d) $3\sqrt{7} - 7$

$(x+7) + \frac{1}{(x+7)} = 7$

$(x+7) - \frac{1}{(x+7)} = 3\sqrt{5}$

14. If $x^2 + \frac{1}{x^2} = 6$, and $0 < x < 1$, what is the value of $x^4 - \frac{1}{x^4}$?

यदि $x^2 + \frac{1}{x^2} = 6$, और $0 < x < 1$, तो $x^4 - \frac{1}{x^4}$ का मान क्या है?

SSC CGL 2023 PRE

(a) ~~$24\sqrt{2}$~~ (b) $-24\sqrt{2}$

(c) $-12\sqrt{10}$ (d) ~~$12\sqrt{10}$~~

$x < \frac{1}{x}$

$\left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$

$\downarrow \qquad \downarrow$

$6 \times -\sqrt{6^2 - 4}$

15. If $x^2 - 16x + 59 = 0$, then find the value of $(x-6)^2 + \frac{1}{(x-6)^2}$?

यदि $x^2 - 16x + 59 = 0$ है, तो $(x-6)^2 + \frac{1}{(x-6)^2}$ का मान ज्ञात कीजिए

(a) 14
(b) 18
(c) 16
(d) 20

$y^2 + \frac{1}{y^2} = 4^2 + 2$

$y^2 + 12y + 36 - 16y - 96 + 59 = 0$

$y^2 - 4y - 1 = 0$

$y - \frac{1}{y} = 4$

माना $x - 6 = y$

$\Rightarrow x = y + 6$

16. If x2 + 2x – 7 = 0 then $(x+4)^4 + \frac{1}{(x+4)^4} = ?$

यदि $x^2 + 2x - 7 = 0$ तो $(x+4)^4 + \frac{1}{(x+4)^4} = ?$

(A) 527
(B) 2207
(C) 1154
(D) 1286

ElectionWay
Test RanKING

If $x + \frac{1}{x} = K$, then $x^3 + \frac{1}{x^3} = K^3 - 3K$

If $x - \frac{1}{x} = K$, then $x^3 - \frac{1}{x^3} = K^3 + 3K$

SelectionWay
Test RanKING

If $x \oplus \frac{1}{x} = \sqrt{t}$, then $x^3 + \frac{1}{x^3} = (t-3)\sqrt{t}$

If $x - \frac{1}{x} = \sqrt{t}$, then $x^3 - \frac{1}{x^3} = (t+3)\sqrt{t}$

if $\frac{z^5}{7} \oplus \frac{7}{z^5} = \sqrt{13}$

तो $\frac{z^{15}}{343} + \frac{343}{z^{15}} = 10\sqrt{13}$ ✓

If $Ax + \frac{B}{Cx} = K$, then

$$A^3x^3 + \frac{B^3}{C^3x^3} = K^3 - 3K \times A \times \frac{B}{C}$$

If $Ax - \frac{B}{Cx} = K$, then

$$A^3x^3 - \frac{B^3}{C^3x^3} = K^3 + 3K \times A \times \frac{B}{C}$$

17. If $x^{0.18} + \frac{1}{x^{0.18}} = 8$, then find $x^{0.54} + \frac{1}{x^{0.54}}$?

यदि $x^{0.18} + \frac{1}{x^{0.18}} = 8$ है, तो $x^{0.54} + \frac{1}{x^{0.54}}$ ज्ञात कीजिये?

(a) 488
(b) 536
(c) 523
(d) 62

$8^3 - 3 \times 8$

18. If $x + \frac{1}{x} = 79$, then find $x\sqrt{x} + \frac{1}{x\sqrt{x}}$?

यदि $x + \frac{1}{x} = 79$ है, तो $x\sqrt{x} + \frac{1}{x\sqrt{x}}$ ज्ञात कीजिये?

(a) 702
(b) 716
(c) 756
(d) 727

$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{81} = 9$

$\sqrt{x}^3 + \frac{1}{\sqrt{x}^3} = 9^3 - 3 \times 9$

19. If $a = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$ and $b = \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$, then find $\frac{a^2}{b} + \frac{b^2}{a} = ?$

यदि $a = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$ और $b = \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ है, तो $\frac{a^2}{b} + \frac{b^2}{a} = ?$

(a) 17498 (b) 17550

(c) 17654 (d) 17576

$\frac{a^3+b^3}{ab} = a^3 + \frac{1}{a^3}$

$= 26^3 - 3 \times 26$

$\therefore 6 - 8$

$a \cdot b = 1$

$b = \frac{1}{a}$

$a + \frac{1}{a} = \frac{2 \times (7+6)}{(7-6)} = 26$

20. If $x^2 + 6x + 1 = 0$, then the value of $(x + 6)^3 + \frac{1}{(x+6)^3} = ?$

यदि $x^2 + 6x + 1 = 0$ है, तो $(x + 6)^3 + \frac{1}{(x+6)^3}$ का मान ज्ञात करें।

(a) 245
(b) 216
(c) 186
(d) 198

$= 6^3 - 3 \times 6$

$x(x+6) = -1$

$x = \frac{-1}{(x+6)}$

$(x+6) + \frac{1}{(x+6)} = 6$

21. If $x(x-3) = -1$, then find $x^5(x^2-7)(x^3-18)$?

यदि $x(x-3) = -1$ है, तो $x^5(x^2-7)(x^3-18)$ ज्ञात कीजिये?

(a) 0 (b) +1

(c) 2 (d) -1

$$x + \frac{1}{x} = 3$$

$$x^2 + \frac{1}{x^2} = 7 \Rightarrow$$

$$x^2(x^2-7) \times x^3(x^3-18)$$

$$\downarrow \qquad \downarrow$$

$$-1 \times -1$$

22. If $x + \frac{1}{3x} = 5$, then the value of $27x^3 + \frac{1}{x^3}$ will be:

यदि $x + \frac{1}{3x} = 5$ है, तो $27x^3 + \frac{1}{x^3}$ का मान ज्ञात करें।

(a) 3240
(b) 3024
(c) 3042
(d) 3420

23. If $\left(3x^2 + \frac{1}{2x^2}\right) = 2.5$, what is the value of $\left(27x^6 + \frac{1}{8x^6}\right)$?

यदि $\left(3x^2 + \frac{1}{2x^2}\right) = 2.5$ है, तो $\left(27x^6 + \frac{1}{8x^6}\right)$ का मान क्या होगा?

(a) 5.125 (b) 6.275

(c) 3.325 (d) 4.375

$2.5^3 - 3 \times 2.5 \times 3 \times \frac{1}{2}$

$= 2.5 \times (6.25 - 4.5)$

$= 2.5 \times 1.75$

24. If $a - \frac{1}{a-5} = 18$, then the value of $(a-5)^3 - \frac{1}{(a-5)^3}$ is:

यदि $a - \frac{1}{(a-5)} = 18$ है, तो $(a-5)^3 - \frac{1}{(a-5)^3}$ का मान क्या होगा?

(a) 2236
(b) 2168
(c) 2239
(d) 2201

$a - 5 = 13$

$= 13^3 + 3 \times 13$

25. If $4p - \frac{5}{2p} = 9, p > 0$, then find the value of $64p^3 + \frac{125}{8p^3}$?

यदि $4p - \frac{5}{2p} = 9, p > 0$, है, तो $64p^3 + \frac{125}{8p^3}$ का मान ज्ञात कीजिये?

(a) 1001
(b) 1298
(c) 900
(d) 990

$1331 - 3 \times 11 \times 4 \times \frac{5}{2}$

$= 1001$

$4p + \frac{5}{2p} = \sqrt{9^2 + 4 \times 4 \times \frac{5}{2}} = 11$

26. If $x^2 - 4x - 3 = 0$, then what is the value of $\frac{\left(x^4 - \frac{27}{x^2}\right)}{(x^2+4x-3)}$?

यदि $x^2 - 4x - 3 = 0$ है, तो $\frac{\left(x^4 - \frac{27}{x^2}\right)}{(x^2+4x-3)}$ का मान क्या होगा?

(a) $9\frac{1}{4}$

(b) $10\frac{1}{2}$

(c) $9\frac{1}{8}$

(d) $12\frac{1}{2}$

$x - \frac{3}{x} = 4$

$\frac{x^3 - \frac{27}{x^3}}{x + 4 - \frac{3}{x}}$

27. If $x^2 - 4x + 1 = 0$ then find $(x^3 - 26)^2 = ?$

यदि $x^2 - 4x + 1 = 0$ तो $(x^3 - 26)^2$ ज्ञात कीजिये

(a) 545 (b) 675

(c) 625 (d) 91.125

$x + \frac{1}{x} = 4$

$x^3 + \frac{1}{x^3} = 52$

$x^6 - 52x^3 + 676$

$x^3(x^3 - 52) + 676$

-1

28. If $\frac{1}{a^4} + a^4 = 50, a > 0$, then find the value of $a^3 + \frac{1}{a^3}$?

यदि $\frac{1}{a^4} + a^4 = 50, a > 0$, है तो $a^3 + \frac{1}{a^3}$ का मान क्या है?

a) $\sqrt{2(1+\sqrt{13})}(-1-2\sqrt{13})$

b) $\sqrt{2(1-\sqrt{13})}(-1+2\sqrt{13})$

c) $\sqrt{2(1+\sqrt{13})} \oplus (-1+2\sqrt{13})$

d) $\sqrt{2(1+\sqrt{13})}\cdot(-1+2\sqrt{13})$

$a^2 + \frac{1}{a^2} = \sqrt{52}$

$a + \frac{1}{a} = \sqrt{2\sqrt{13}+2}$

if, $a + \frac{1}{a} = \sqrt{t}$

$a^3 + \frac{1}{a^3} = (t-3)\cdot\sqrt{t}$

29. If a2 + a = 13, then find the value of $(a + 4)3 - \frac{1}{(a+4)^3}$

यदि $a^2 + a = 13$, है तो $(a + 4)^3 - \frac{1}{(a+4)^3}$ का मान ज्ञात करे-

[A] 364
[B] 540
[C] 440
[D] 420

$x^2 - 8x + 16 + x - 4 = 13$

$x^2 - 7x - 1 = 0$

$x - \frac{1}{x} = 7$

Let $a + 4 = x$

$a = x - 4$

$x^3 - \frac{1}{x^3} = 7^3 + 3 \times 7$