



$$\rightarrow (a + b + c)^2 = a^2 + b^2 + c^2 + 2(\underline{ab + bc + ca})$$

$$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$$

$$\checkmark \quad ab + bc + ca = \frac{(a + b + c)^2 - (a^2 + b^2 + c^2)}{2}$$

$$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$



$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

#



If a, b, c are in A.P with common diff d then
यदि a, b, c समांतर श्रेणी में है, तथा सार्व अंतर d है,



तो

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [d^2 + d^2 + (2d)^2]$$

$$= 3d^2$$

$$= 3(\text{common diff})^2$$



$$\text{If, } a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\text{or } a^2 + b^2 + c^2 = ab + bc + ca$$

$$\checkmark \text{ then } a = b = c$$



$$a^3 + b^3 + c^3 - 3abc =$$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= (a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$$

$$= \frac{1}{2}(a + b + c)[3(a^2 + b^2 + c^2) - (a + b + c)^2]$$



$$\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2} = \frac{a+b+c}{2}$$



$$a^3 + b^3 + c^3 - 3abc = (a + b + c) [(a + b + c)^2 - 3(ab + bc + ca)]$$

↓

$$\text{If } a + b + c = 0$$

↓

तो

$$a^3 + b^3 + c^3 - 3abc = 0$$

or

$$a^3 + b^3 + c^3 = 3abc$$



$$a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c) [(a - b)^2 - (b - c)^2 + (c - a)^2]$$

If $a^3 + b^3 + c^3 - 3abc = 0$, then

$$a^3 + b^3 + c^3 = 3abc$$

If a, b, c are distinct integers

$$a + b + c = 0$$

If a, b, c are +ve integers

$$a = b = c$$



If a, b, c are in A.P then

$$a^3 + b^3 + c^3 - 3abc = 9 \times b \times d^2 \quad (d = \text{common difference})$$

$$9 \times \text{Middle Number} \times (\text{common difference})^2$$



1. Write the expanded form of $(8a + 5b + 9c)^2$.

$(8a + 5b + 9c)^2$ को विस्तारित रूप में लिखिए।

[RRB JE 2024]

- A) $64a^2 + 25b^2 + 81c^2 + \cancel{84ab} + 90bc + 144ac$
- B) $64a^2 + 25b^2 + 81c^2 + 80ab + \cancel{85bc} + 144ac$
- C) $64a^2 + 25b^2 + 81c^2 + 80ab + 90bc + 144ac$
- D) $64a^2 + 25b^2 + 81c^2 + 80ab + 90bc + \cancel{154ac}$



2. If $(x + y + z) = 23$ and $x^2 + y^2 + z^2 = 179$, then find the value of $(xy + yz + zx)$.
यदि $(x + y + z) = 23$ और $x^2 + y^2 + z^2 = 179$ है, तो $(xy + yz + zx)$ का मान ज्ञात कीजिए।

(SSC SELECTION POST XII HIGHER SECONDARY LEVEL)

[A] 225

[C] 175 ✓

[B] 280

[D] 350

$$\begin{aligned} & \downarrow \\ & \frac{23^2 - 179}{2} \\ & = 175 \end{aligned}$$



3. If $a + b + c = 13$ and $ab + bc + ca = 45$, find $a^2 + b^2 + c^2$.

यदि $a + b + c = 13$ और $ab + bc + ca = 45$ है, तो $a^2 + b^2 + c^2$ का मान ज्ञात कीजिए।

[A] 79

[C] 57

[B] 65

[D] 85

$$\begin{aligned} & \downarrow \\ & = 13^2 - 2 \times 45 \\ & = 79 \end{aligned}$$





4. If $a^2 + b^2 + c^2 = 6.25$ and $(ab + bc + ca) = 0.52$, what is the value of $(a+b+c)$, if $(a+b+c) < 0$?
- यदि $a^2 + b^2 + c^2 = 6.25$ और $(ab + bc + ca) = 0.52$ है, तो $(a+b+c)$ का मान ज्ञात करें, यदि $(a+b+c) < 0$ है?

(a) ~~± 2.7~~
(c) -2.8

✓ (b) -2.7
(d) ~~± 2.8~~

$$= -2.7$$

$$(a+b+c)^2 = 6.25 + 2 \times 0.52 = 7.29$$



5. If $\frac{a^2+b^2+c^2-1024}{ab-bc-ca} = -2$ & $a+b=5c$, where $c>0$, then the value of c is?

यदि $\frac{a^2+b^2+c^2-1024}{ab-bc-ca} = -2$ & $a+b=5c$ है, जहाँ $c>0$, तो c का मान है?

[A] 8

[C] 12

[B] 4

[D] 5

$$a^2+b^2+c^2+2ab-2bc-2ca=1024$$

$$(a+b-c)^2=1024$$

$$a+b-c=32$$

$$4c=32$$

$$c=8$$

6. **The square root of the expression $\frac{1}{abc}(a^2 + b^2 + c^2) + 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$?**

व्यंजक $\frac{1}{abc}(a^2 + b^2 + c^2) + 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ का वर्गमूल है

[A] $\frac{a+b+c}{abc}$

[B] $\sqrt{a} + \sqrt{b} + \sqrt{c}$

[C] $\sqrt{\frac{bc}{a}} + \frac{ca}{b} + \sqrt{\frac{ab}{c}}$

[D] $\sqrt{\frac{a}{bc}} + \sqrt{\frac{b}{ca}} + \sqrt{\frac{c}{ab}}$

$$\begin{aligned} &= \sqrt{\frac{(a+b+c)^2}{abc}} \\ &= \frac{a+b+c}{\sqrt{abc}} \end{aligned}$$



7. If $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx = (Ax + y + Bz)^2$, then the value of $(A^2 + B^2 - AB)$ is:

यदि $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx = (Ax + y + Bz)^2$ है, तो $(A^2 + B^2 - AB)$ का मान क्या है?

[A] 16

[C] 6

[B] 14

[D] 18

$$= 2 + 8 + 4$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

$$A = -\sqrt{2}$$

$$B = 2\sqrt{2}$$



8. **If $a + b + c = 6$ & $(a^2 + b^2 + c^2) = 14$ & $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{6}$, then find abc ?**

यदि $a + b + c = 6$ & $(a^2 + b^2 + c^2) = 14$ & $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{6}$ है, तो **abc** ज्ञात कीजिये?

[A] 12

[C] 1

☒ [B] 6

[D] 11





9.

If $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \sqrt{3}$ and $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$, then find the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$

(Where x, y, z, a, b, c are non zero numbers)

यदि $\left(\frac{x}{a}\right) + \left(\frac{y}{b}\right) + \left(\frac{z}{c}\right) = \sqrt{3}$ और $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ तो $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ का मान ज्ञात कीजिए।

(जहाँ x, y, z, a, b, c शून्येतर संख्याएँ हैं)

$$u^2 + v^2 + w^2 = \sqrt{3}^2 - 2 \times 0 = 3$$

[A] 0

[B] $\sqrt{3}$

[C] 3

[D] 6

$$u + v + w = \sqrt{3} \quad \frac{1}{u} + \frac{1}{v} + \frac{1}{w} = 0$$

$$\Rightarrow u^2 + v^2 + w^2 = 0$$



10. If $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 11$ and $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$ then find the value of $\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}$?

यदि $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 11$ और $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$ है, तो $\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}$ का मान ज्ञात कीजिए।

RRB NTPC 2021

[A] 105

[C] 137

[B] 19

[D] 75

सूत्र
 a, b, c

$$= \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \right)^2 - 2 \left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b} \right)$$

$$= 11^2 - 2 \times 8$$
$$= 105$$

11. If $(a + b + c) = 17$, and $(a^2 + b^2 + c^2) = 115$, find the value of $(a + b)^2 + (b + c)^2 + (c + a)^2$.

यदि $(a+b+c) = 17$, और $(a^2 + b^2 + c^2) = 115$, तो $(a+b)^2 + (b+c)^2 + (c+a)^2$ का मान ज्ञात कीजिए।

[A] 402

[B] 404

[C] 408

[D] 394

2 eqⁿ

a, b & c

put c=0

$$ab = \frac{17^2 - 115}{2} \\ = 87$$

$$= 2 \cdot (a^2 + b^2 + ab)$$

$$= 2 \times (115 + 87)$$

$$= 404$$



12. If $(a + b + c) = 17$ and $(a^2 + b^2 + c^2) = 117$, what is the value of $(a - b)^2 + (b - c)^2 + (c - a)^2$?

यदि $(a + b + c) = 17$ और $(a^2 + b^2 + c^2) = 117$, तो $(a - b)^2 + (b - c)^2 + (c - a)^2$ का मान क्या होगा?

[A] 57

[B] 72

[C] 42

☒ [D] 62



13. If, $(a-b)^2 + (b-c)^2 + (c-a)^2 = 6$ and $a^2 + b^2 + c^2 = 29$, then what is $(a+b+c)$ equal to?
यदि $(a-b)^2 + (b-c)^2 + (c-a)^2 = 6$ और $a^2 + b^2 + c^2 = 29$ है, तो $(a+b+c)$ किसके बराबर है?

(UPSC CDS - II 2025)

[A] ± 9
[C] ± 6

[B] ± 8
[D] ± 3

put $c=0$

$$(a^2 + b^2 - ab) = 3$$

$$29 - ab = 3$$

$$ab = 26$$

$$\begin{aligned} &= \pm \sqrt{a^2 + b^2 + 2ab} \\ &= \pm \sqrt{29 + 52} \\ &= \pm 9 \end{aligned}$$

14. If a, b, c are non-zero real numbers and $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = 0$, then find the value of $\frac{6(a+b+c)+2c}{(a+b+c)}$?

यदि a, b, c गैर-शून्य वास्तविक संख्याएं हैं और $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = 0$, तो

$\frac{6(a+b+c)+2c}{(a+b+c)}$ का मान ज्ञात कीजिये?

[A] 1
[C] 6

[B] 8
[D] 7

$$\frac{14c}{2c} = 7$$

$$(a+b-c)^2 = 0$$
$$a+b=c$$



15. If $x \neq 0$, $y \neq 0$ and $z \neq 0$ and $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$, then the relation among x , y , z is:

[A] $x + y + z = 0$

[C] $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

[B] $x + y = z$

[D] $x = y = z$

$\Rightarrow \frac{1}{x} = \frac{1}{y} = \frac{1}{z}$



16. If a, b, c are non-zero real numbers, $abc = 27$ and $a^2 + b^2 + c^2 = ab + bc + ca$ then the value of $\frac{a^2 + 2b^2}{5c} = ?$

यदि a, b, c गैर-शून्य वास्तविक संख्या हैं, $abc = 27$ और $a^2 + b^2 + c^2 = ab + bc + ca$ तब $\frac{a^2 + 2b^2}{5c}$ का मान है ?

[A] 1.6

[C] 1.8

[B] 1.5

[D] 1.4

$$\Rightarrow a = b = c = 3$$

$$\begin{aligned} &= \frac{3a}{5} \\ &= 1.8 \end{aligned}$$



17. If $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $\frac{11a^4 + 13b^4 + 15c^4}{16a^2b^2 + 19b^2c^2 + 17c^2a^2}$ is:

यदि $a^2 + b^2 + c^2 = ab + bc + ca$, तो $\frac{11a^4 + 13b^4 + 15c^4}{16a^2b^2 + 19b^2c^2 + 17c^2a^2}$ का मान क्या है?

[A] $1\frac{3}{4}$
[C] $1\frac{1}{3}$

[B] $\frac{1}{4}$
[D] $\frac{3}{4}$ = $\frac{39}{52}$

$\Rightarrow a = b = c$

$1:1:1$

18. If $x + y + z = 9\sqrt{5}$ and $x^2 + y^2 + z^2 = 135$, then $\frac{x^2 + y^2 + z^2}{x^2 + y^2 + z^2} = ?$

[A] $\frac{46\sqrt{5}}{15}$
[C] $\frac{46\sqrt{5}}{9}$

[B] $\frac{46\sqrt{5}}{5}$
[D] $\frac{46\sqrt{3}}{9}$

$$= \frac{1 + \frac{46\sqrt{5}}{9}}{\frac{46\sqrt{5}}{9}} = \frac{46\sqrt{5}}{3\sqrt{5} \times \sqrt{5}}$$

special case

$$xy + yz + zx = \frac{(9\sqrt{5})^2 - 135}{2} = 135$$

$$x^2 + y^2 + z^2 = xy + yz + zx \\ \Rightarrow x = y = z = 3\sqrt{5}$$



19. If $a=97.5$, $b=100$, $c=102.5$, then find $a^2 + b^2 + c^2 - ab - bc - ca = ?$

[A] $\frac{81}{4}$

[B] $\frac{70}{4}$

[C]

$\frac{75}{4}$

+2.5

[D]

15

$\downarrow$
 $= 3 \times \left(\frac{5}{2}\right)^2$





20. Find $6637^2 + 6640^2 + 6642^2 - 6637 \times 6640 - 6640 \times 6642 - 6642 \times 6637$?

[A] 13

[B] 17

[C] 21

[D] 19

$$= \frac{1}{2} \cdot [9 + 4 + 25]$$
$$= 19$$





21. If $a - b = 2$ & $b - c = 3$, then find the value of $a^2 + b^2 + c^2 - ab - bc - ca$?

यदि $a - b = 2$ & $b - c = 3$ है, तो $a^2 + b^2 + c^2 - ab - bc - ca$ का मान ज्ञात किजिए?

[A] 20

✓ [C] 19 add

[B] 18

[D] 31

$$a - c = 5$$

$$\frac{1}{2} [4 + 9 + 25]$$

